A Level Set Based Predictor-Corrector Algorithm for Vessel Segmentation

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***Abstract*— Vessel segmentation is an essential task in many computer-aided medical systems. However, the topology complexity of vascular structures and the intensity inhomogeneity of angiogram make it a challenging problem. We propose a level set based predictor-corrector algorithm to meet these challenges. In the predictor step, the overall contour of vessel structures is delineated by *piecewise constant* (PC) model, which is insensitive to the initial contour and adaptive to the complex morphological variations of vessel structures. In the corrector step, the segmented results are refined by an improved *local binary fitting* (LBF) model, which can efficiently deal with intensity inhomogeneity in the angiogram, especially in the distal part of the vessels. Compared to original LBF model, our approach can avoid the emergence of new contour in non-vascular regions. The proposed algorithm takes both global and local information into consideration and combines the advantages of PC model and LBF model. Experimental results on MRA images demonstrate the feasibility of our algorithm.**

I. Introduction

Vessel segmentation is an essential task in many computer-aided medical systems. It is usually a prerequisite to the implementation of other modules such as vascular modeling, biomechanical and hemodynamical analysis and surgical planning and simulation. However, automatic vessel segmentation is a challenging task in clinical practice because of the high degree of topological complexity of vascular structures and the inevitable intensity inhomogeneity in almost all imaging modalities, including 3D rotational angiography (3DRA), computer tomography angiography (CTA) and magnetic resonance angiography (MRA). The intensity inhomogeneity may cause more difficulties in the distal part of vessels, where the contrast between blood vessels and surrounding tissues is usually lower than other regions due to signal loss or other technical limitations.

Recent years, a lot of algorithms have been proposed to meet these challenges [1]. In this work, we focus on level set based method, which is a suitable approach for vessel segmentation, as it can efficiently track interfaces and shapes of complex topology and allow for simple calculation of geometrical properties of moving surfaces. Chan and Vese's *piecewise constant* (PC) model [2] is one of initial region-based level set models, which has great flexibility of handling the large morphological variations and insensitive to the initial contour. However, the main disadvantage of PC model is that it tends to rely on intensity homogeneity in each of the regions to be segmented.

To deal with intensity inhomogeneity, Li *et al.* [3] proposed a new model, namely *local binary fitting* (LBF) model, which introduces a kernel function to define a local binary fitting energy in a variational formulation, so that local intensity information can be embedded into a region-based active contour model. Although it can handle intensity inhomogeneity in some simple images, the main disadvantage of this model is that new contours of surrounding tissues may emerge during the curve evolution because it only takes local intensity information into account. In addition, the segmented results are usually sensitive to initial contour.

In this paper, we propose an improved LBF model by employing a new Heaviside function to avoid the emergence of new contour of non-vascular regions during the evolution. A novel predictor-corrector algorithm is further developed by combining PC model and the improved LBF model. Our approach not only can deal with the intensity inhomogeneity in angiogram but is adaptive to the high degree of topology complexity of vascular structures and insensitive to initial contour and other noises. Experiments demonstrate that our algorithm can achieve better results than PC model and original LBF model. In the following section, we first review some existing algorithms for vessel extraction. In section III, we give the implementation details of the proposed method. In section IV, we provide experimental results. Finally, we give a short discussion and draw conclusion in section V.

II. Related Work

A lot of vessel segmentation algorithms have been proposed recent years. The simplest method is threshold based algorithm [4] where an iso-surface is directly extracted based on image intensity. However, it may fail in many cases where the image intensity is inhomogeneous and tissues with similar intensity of vessels exist. Region growing approaches [4], [5] are also proposed for vessel segmentation. The disadvantage of it is that the growth process may leak into non-vascular structures during the growing process due to the complex topology of vascular structures.

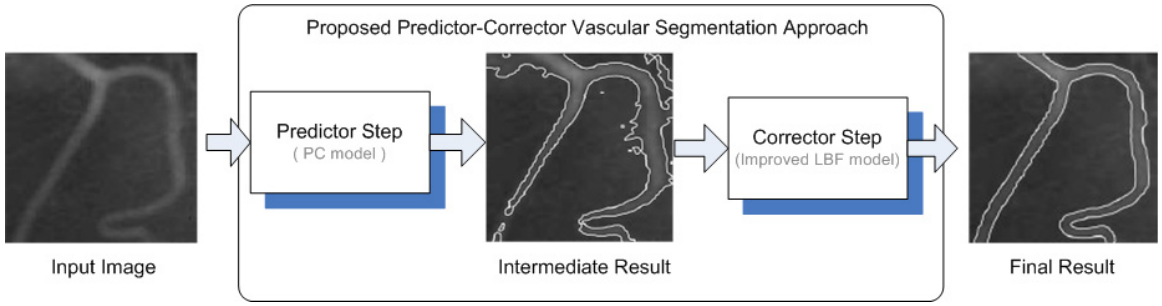

Fig. 1. Overview of proposed method.

Deformable models [6], [7] are also employed for vessel segmentation. In these algorithms, an initial surface is first constructed to represent vascular topology. Then, the surface deforms to approach the boundaries of vessels under internal elastic forces between neighboring nodes and external forces derived from local image intensity gradient. However, this method tends to rely on the correct topology of vascular structures as initial condition, which is usually not easy to be obtained automatically, especially in the cases that the topology is complex.

Extracting vessels based on level set methods has been an active research topic recent years. In order to overcome above-mentioned difficulties of PC model in segmenting images with intensity inhomogeneity, *piecewise smooth* (PS) model [8], [9] are proposed. However, several computation-intensive tasks in the algorithm make it computationally expensive. Some improved PS models [10], [11], [12] have taken the local information into consideration to further remove the segmented artifacts caused by intensity inhomogeneity, but the computational performance is still a problem.

Recently, Wang *et al.* proposed an algorithm that combined the global model and the LBF model for brain segmentation [13]. To the best of our knowledge, this is the most similar work to this paper. However, the method is just a simple combination of the two models and fails to solve some disadvantages of LBF model, such as the undesired emergence of new contours at strong object boundaries and the sensitivity to initialization.

III. Implementation

The proposed model employs a predictor-corrector scheme to tackle the intensity inhomogeneity exists in vacular medical images. Figure 1 shows an overview of the proposed approach. In the prediction step, a global level set evolution tries to predict a rough object boundary which may contain errors. Then, in the corrector step, this roughly segmented result is used as initial contour and subjected to a local level set evolution. An improved LBF model is proposed to precisely extract boundaries at blurry regions or regions with intensity inhomogeneity. We will now introduce the related details in these two steps in the following of this section.

A. The Predictor Step : PC model

In the predictor step, PC model is employed to obtain an initial contour that well delineates the overall vascular topology. For an image $I(x,y)$, the global energy functional $E_G(C)$ is

$$\begin{aligned} E_G(C) &= \lambda_1 \int_{in(C)} |I(x)-c_1|^2 dx \\ &+ \lambda_2 \int_{out(C)} |I(x)-c_2|^2 dx + \nu|C| \end{aligned} \quad (1)$$

In the level set formulation, with $C=\{(x,y)|\phi(x,y)=0\}$ and ϕ being a level set function, $E_G(C)$ can be expressed as a function of ϕ:

$$\begin{aligned} E_G(\phi) = \lambda_1 \int_\Omega |I(x)-c_1|^2 H(\phi) dxdy \\ +\lambda_2 \int_\Omega |I(x)-c_2|^2 (1-H(\phi)) dxdy + \nu \int_\Omega |\nabla H(\phi)| \end{aligned} \quad (2)$$

where λ_1,$\lambda_2 > 0$, $\nu \geq 0$ are fixed parameters, H is the Heaviside function, $in(C)$ and $out(C)$ represent the region inside and outside of the contour C, and c_1 and c_2 are two constants that approximate the image intensity in $in(C)$ and $out(C)$, respectively.

Minimizing the energy function in Eq.(2) with respect to ϕ, we obtain:

$$\frac{\partial \phi}{\partial t} = \delta_\epsilon(\phi)[\nu div(\frac{\nabla \phi}{|\nabla \phi|}) - \lambda_1(I-c_1)^2 + \lambda_2(I-c_2)^2)] \quad (3)$$

where δ is the Dirac delta function, and c_1 and c_2 are defined by

$$c_1 = \frac{\int I(x)H(\phi(x))dx}{\int H(\phi(x))dx}$$

$$c_2 = \frac{\int I(x)(1-H(\phi(x)))dx}{\int (1-H(\phi(x)))dx} \quad (4)$$

The prediction segmentation results ϕ^p can be obtained by solving the Eq.(3). We employ finite difference scheme to approximate the partial derivatives of ϕ with respect to x and y in the calculation. More implementation details can be found in [2].

B. The Corrector Step: an Improved LBF Model

After the prediction step, blood vessel with high intensity contrast at boundaries can be segmented. Usually, it works well if blurry or noisy boundaries do not exist. Figure 1 shows such an example of the intermediate segmented result. We can find the extracted segment has consistent shape as the vessels globally, while some

artifacts can be found in some parts of the boundary when looking closely. Blurry boundaries (usually in the distal part of vessels) need to be further corrected by taking local information into consideration. Therefore, in the corrector step, we use an improved LBF model to refine the segmented results obtained from the prediction step. The local energy can be formulated as

$$E_L(\phi) = \varepsilon^{LBF}(\phi, f_1, f_2) + \mu\mathcal{P}(\phi) + \nu\mathcal{L}(\phi) \tag{5}$$

where μ and ν are nonnegative constants. The three terms in the right of the equation are the local data fitting term, the level set regularization term and the arc length term respectively. The arc length term is proposed to smooth the zero level set contour which is computed by

$$\mathcal{L}(\phi) = \int |\nabla H(\phi(x))| dx \tag{6}$$

The level set regularization term, which is proposed in [14] , serves to maintain the regularity of the level set function:

$$\mathcal{P}(\phi) = \int \frac{1}{2}(|\nabla H(\phi(x))| - 1)^2 dx \tag{7}$$

It penalizes the deviation of the function ϕ from a signed distance function. Thus, the level set function needs no re-initialization on the evolution.

The local data fitting term $\varepsilon^{LBF}(\phi, f_1, f_2)$ is defined as:

$$\begin{aligned} & \varepsilon^{LBF}(\phi, f_1, f_2) \\ = \; & \lambda_1 \int\int K_\sigma(x-y)|I(y) - f_1(x)|^2 H(\phi(y)) dy dx \\ + \; & \lambda_2 \int\int K_\sigma(x-y)|I(y) - f_2(x)|^2 \\ & (1 - H(\phi(y))) dy dx \end{aligned} \tag{8}$$

where λ_1 and λ_2 are two positive constants, H is Heaviside function, and K_σ is a Gaussian kernel with standard deviation σ. The local data fitting energy is dominated by the intensities $I(y)$ in a neighborhood of x due to the localization property of the kernel function, and this localization property enables the LBF model to deal with intensity inhomogeneity. The $f_1(x)$ and $f_2(x)$ are two spatially varying fitting functions which are introduced to approximate the local intensities on the two sides of the contour, and they are defined by

$$f_1(x) = \frac{K_\sigma(x) * [H(\phi(x))I(x)]}{K_\sigma(x) * H(\phi(x))}$$

$$f_2(x) = \frac{K_\sigma(x) * [(1 - H(\phi(x)))I(x)]}{K_\sigma(x) * [1 - H(\phi(x))]} \tag{9}$$

Minimization of the energy function in Eq.(5) with respect to ϕ is achieved by solving the gradient descent flow equation :

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= -\delta(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \nu\delta(\phi) div\big(\frac{\nabla\phi}{|\nabla\phi|}\big) \\ &+ \mu\big(\nabla^2\phi - div\big(\frac{\nabla\phi}{|\nabla\phi|}\big)\big) \end{aligned} \tag{10}$$

where δ is the Dirac delta function, and $e_i = \int_\Omega K_\sigma(y-x)|I(x) - f_i(y)|^2 dy, i = 1, 2.$

When solving the corrector model in the formulation (10), segmented results of the prediction step ϕ^p is used as the initial value of the differential equation.

In the original LBF model, function H is approximated by a smooth function H_ϵ defined by

$$H_\epsilon(x) = \frac{1}{2}[1 + \frac{2}{\pi}\arctan(\frac{x}{\epsilon})] \tag{11}$$

Its derivative δ_ϵ is the smoothed Dirac delta function given as follows,

$$\delta_\epsilon(x) = H'_\epsilon(x) = \frac{1}{\pi}\frac{\epsilon}{\epsilon^2 + x^2} \tag{12}$$

However, when the above approximation is applied to segmentation of vascular images, it may cause emergence of contours at non-vascular regions which possess strong intensity contrast. The main reason behind this phenomena is related to the factor δ_ϵ in the equation. At regions with strong intensity contrast or intensity inhomogeneity, the data fitting term $(\lambda_1 e_1 - \lambda_2 e_2)$ may not be close to zero. Therefore, when δ_ϵ takes small values at the regions far away from the zero level set, the term $\delta_\epsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2)$ will be much larger than zero at these regions and cause new contours appear there.

Although in [15], the authors suggested to use a larger λ_2 than λ_1 to avoid the emergence of new contours far away from the initial contour, such as the skull boundaries. In our case, however, if we choose a larger λ_2 than λ_1, we found that the total iteration will increase significantly to reach the same segmented results. In order to keep the computation efficiency, we introduce a function of H_α, which improves the deficiency caused by H_ϵ, to approximation the function of H.

$$H_\alpha(x) = \begin{cases} 1 & \text{if } x > \alpha \\ 0 & \text{if } x < -\alpha \\ \frac{1}{2}[1 + \frac{x}{\alpha} + \frac{1}{\pi}\sin(\frac{\pi x}{\alpha})] & \text{if } |x| \le \alpha \end{cases} \tag{13}$$

The derivative of $H_\alpha(x)$ is

$$\delta_\alpha = H'_\alpha(x) = \begin{cases} 0 & \text{if } |x| > \alpha \\ \frac{1}{2\alpha}[1 + \cos(\frac{\pi x}{\alpha})] & \text{if } |x| \le \alpha \end{cases} \tag{14}$$

where α is a positive parameter to control the degree to supress emergence of new contours. In our proposed model, even $(\lambda_1 e_1 - \lambda_2 e_2)$ is still large at high contrast regions, δ_α will be zero if it is far away from the zero level set, as $\delta_\alpha(\phi) = 0$ when $|\phi| > \alpha$. Thus, the data fitting term $\delta_\alpha(\phi)(\lambda_1 e_1 - \lambda_2 e_2)$ will also be zero. Therefore, new unexpected contours will not emerge at high contrast regions which is far away from the zero level set.

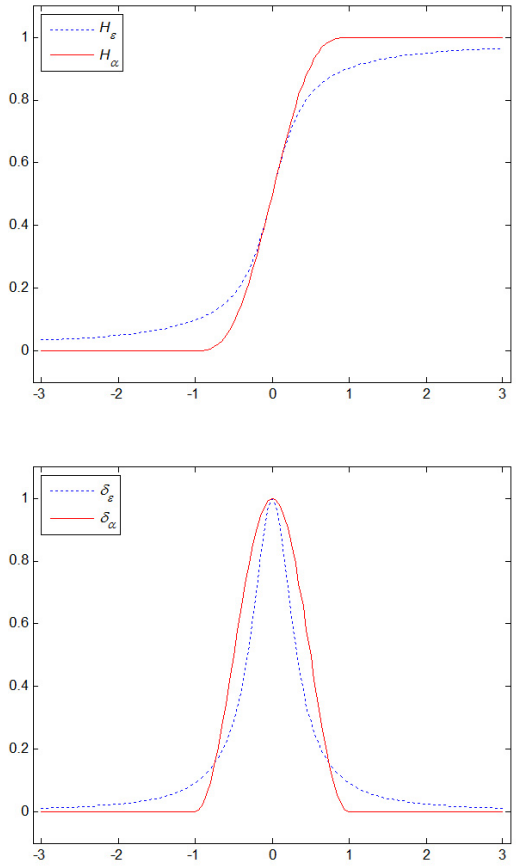

Fig. 2. Two approximations of the Heaviside function H and delta function δ

The advantage of formulating H_α as in Equation 13 can be more easily understood by looking at Fig. 2. We can find that $supp(H_\alpha) \subseteq supp(H_\epsilon)$. Thus, it not only effectively avoids the emergence of new contours, but still keeping $H_\alpha \in C^2$ hold.

IV. Results

A series of experiments have been conducted to validate the feasibility of the proposed modifications in the level set model. First, we employ our method to MRA images with relatively simple vascular structures. The results are shown in Fig. 3 and Fig. 4. In Fig. 3, we employ the following parameters: $\sigma = 3.0, \mu = 1, \nu = 0.002 \times 255 \times 255, \alpha = 0.9$, while in Fig. 4 the corresponding parameters are $\lambda_1 = \lambda_2 = 1.0$, $\lambda_2 = 1.1$ and $\lambda_1 = 1.0$.

In Fig. 3(b), the result using PC model fails to extract the distal parts of vessels, where the contrast between blood vessels and surrounding tissues is relatively low. Meanwhile, the original LBF model shown in Fig. 3(c) also cannot achieve desirable results, new contours of non-vascular tissues emerge. In contrast, our algorithm shows a satisfactory segmentation result in Fig. 3(d). It not only finely extracts the distal parts of vessels but also avoids the emergence of new contours at non-vascular tissues. Similarly, in Fig. 4, one can find from the enlarged regions at the bottom, the proposed method is capable to extract even some fine vessels.

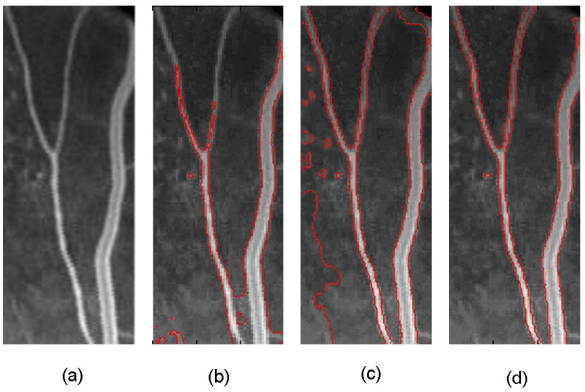

Fig. 3. (a)Original image. (b)The result of PC model. (c)The result of LBF model. (d)The result of our model.

For MRA images with more complex vascular structures as shown in Fig. 5(a), we can compare our result with the two previous methods in Fig. 5. Again, our method can produce tidy segmentation of the whole vascular structure with high degree of topology complexity. All the above examples demonstrate that our algorithm achieve better results than both PC model and LBF model in vascular segmentation. Experiments are also performed on images suffered from noise (Fig. 6) which is common in medical images. Our method can consistently perform well in these examples. It also means that our proposed method has better noise-resistance compared to PC model and LBF model.

V. Conclusion

We have presented a novel level set based predictor-corrector algorithm for vessel segmentation. The proposed algorithm takes both the global and local information into account. In the prediction step, the PC model is applied to obtain the predicted segmentation contour. Then in the corrector step, an improved LBF model is proposed to produce a final segmentation result at the basis of the predicted contour. Our algorithm combines the advantages of the PC model and the LBF model. It can precisely extract the vessel from angiogram without the emergence of new contours of non-vascular tissues. Meanwhile, it is insensitive to initial contour and robust to noise. Experimental results have demonstrated the advantages of our model. Future improvements include further evaluating our algorithm on 3DRA and CTA, implementing it on GPU to achieve better performance and integrating it into some clinical computer-aided medical systems.

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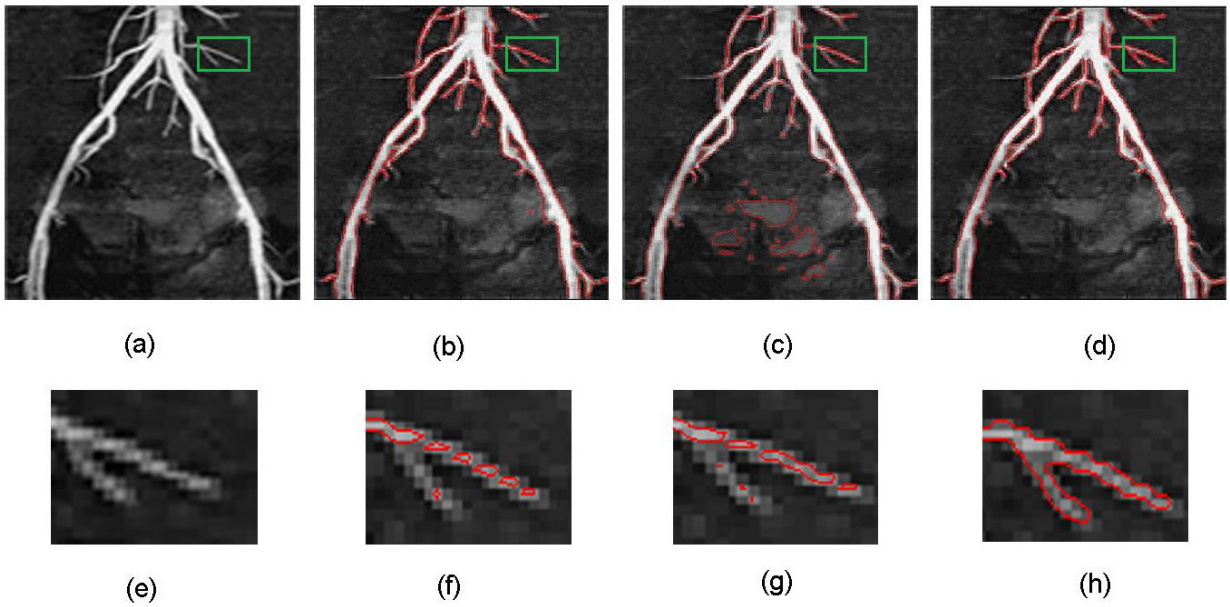

Fig. 4. (a)(e)Original image. (b)(f)The results of PC model. (c)(g)The results of LBF model. (d)(h)The results of our model.

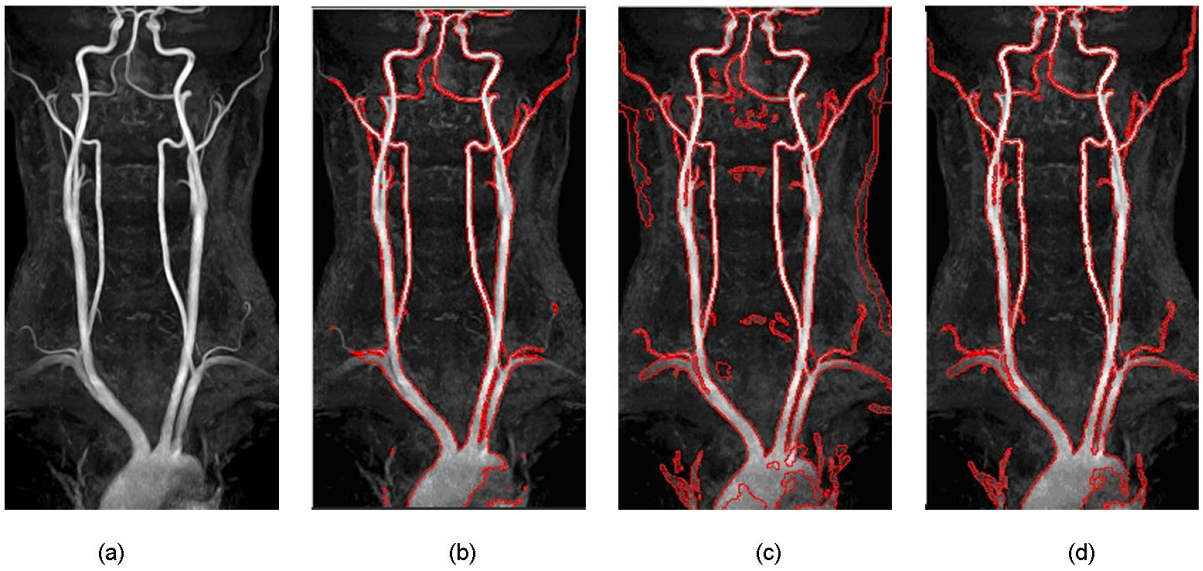

Fig. 5. (a)Original image. (b)The result of PC model. (c)The result of LBF model. (d)The result of our model.

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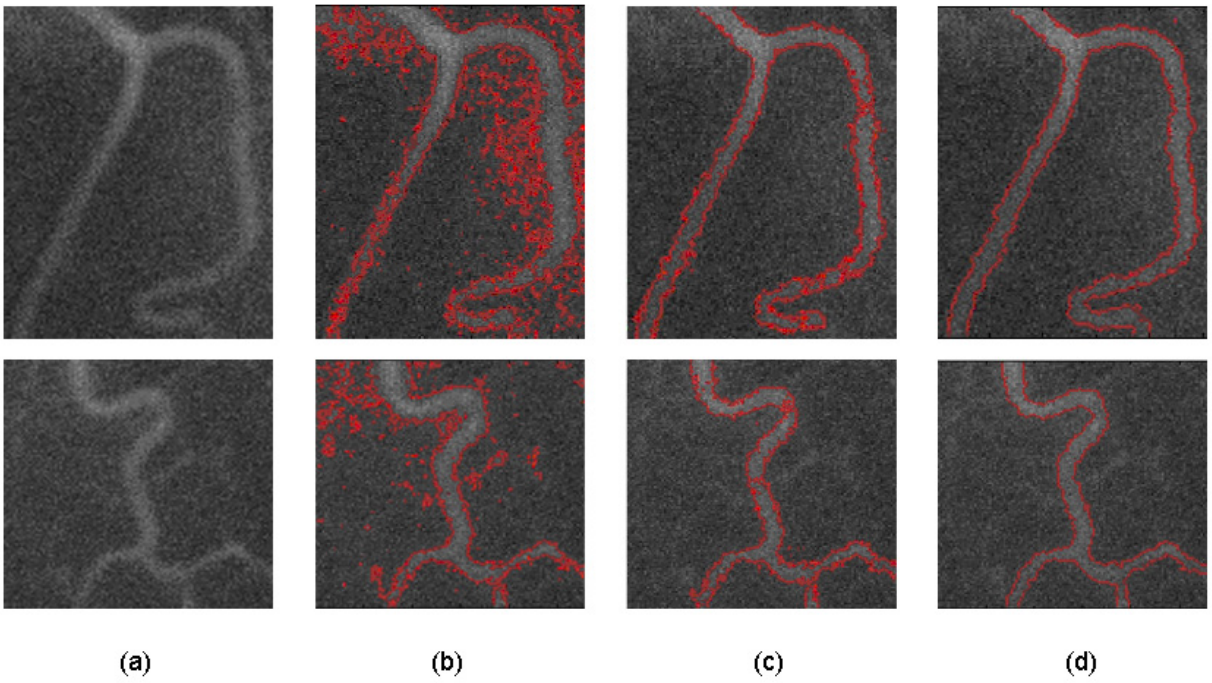

Fig. 6. The noisy images.(a)Original image. (b)The results of PC model. (c)The results of LBF model. (d)The results of our model.

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