

Please provide details and steps of your work!!!

**Name:****Student ID or Lucky No.:**

1. (8 points) Let  $f(x) = x^{1/3}$ , find
  - (a)  $f'(x)$
  - (b) the partition numbers for  $f'(x)$
  - (c) the critical values of  $f(x)$ .
2. (10 points) Let  $f(x) = x^3 - 12x + 1$  on the interval  $[-3, 5]$ . Identify the critical points and find the absolute maximum value and absolute minimum value on the given interval  $[-3, 5]$ .
3. (16 points) Evaluate the following limits
  - (a)  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$
  - (b)  $\lim_{x \rightarrow \infty} \frac{e^{4x}}{x^2}$
  - (c)  $\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4}$
  - (d)  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$
4. (8 points) Use the second-derivative test to find any local extrema for

$$f(x) = x^3 - 6x^2 - 15x + 1$$

5. (10 points) Let  $f(x) = x^3(x - 3) = x^4 - 3x^3$ . Summarize all the pertinent information obtained by applying the graphing strategy to  $f$ , and then sketch the graph of  $f$  as follows.
  - (a) Analyze  $f(x)$ : find the domain, intercepts, symmetry(if exists), asymptotes(if exists).
  - (b) Analyze  $f'(x)$ : find local extrema, increasing and decreasing intervals.
  - (c) Analyze  $f''(x)$ : find inflection points, concavity.
  - (d) Sketch the graph of  $f$ .
6. (15 points) Find each indefinite integral:
  - (a)  $\int 3x^4 dx$
  - (b)  $\int (\frac{2}{x} - 4e^x) dx$
  - (c)  $\int (5\sqrt{x} - \frac{6}{\sqrt{x}}) dx$
  - (d)  $\int \frac{4}{4x+6} dx$
  - (e)  $\int \frac{(\ln x)^2}{x} dx$
7. (8 points) Find the particular antiderivative of the following derivative that satisfies the given initial condition:  
 $F'(x) = 6x^2 - 2x; F(0) = 1$

8. (8 points) Find the derivative or indefinite integral of the following:

(a)  $\frac{d}{dx}(\int e^{x^2} dx)$

(b)  $\int \frac{d}{dx}(\sqrt{4+3x})dx$

9. (5 points) Calculate the Riemann sum  $S_n = \sum_{i=1}^n f(c_i)\Delta x_i$  when  $f(x) = 2x$ ; the partition is  $P_5 : 0 < 0.5 < 1.25 < 1.75 < 2.5 < 3$ ; and sample points are  $c_1 = 0, c_2 = 1, c_3 = 1.75, c_4 = 2, c_5 = 3$ .

10. (12 points) Evaluate the following definite integrals:

(a)  $\int_1^1 (x+2)^9 dx$

(b)  $\int_1^2 x^{1/2} dx$

(c)  $\int_0^1 (1-2x^2) dx$

(d)  $\int_0^1 2xe^{x^2} dx$

**Bonus (20 Points)**

11. (10 points) A homeowner has \$ 160 to spend on building a fence around a rectangular garden. Three sides of the fence will be constructed with wire fencing at a cost of \$ 1 per linear foot. The fourth side will be constructed with wood fencing at a cost of \$ 3 per linear foot. Find the dimensions and the area of the largest garden that can be enclosed with \$ 160 worth of fencing.
12. (10 points) Let  $f(x) = 2x$  on the interval  $[1, 3]$ . Calculate the left Riemann sum  $S_n$ , when partition the interval  $[1, 3]$  into  $n$  subintervals of equal length.