Please provide details and steps of your work!!!

Name:
Student ID:

1. (8 points) Evaluate the following limits
   (a) $\lim_{x\to 1} \frac{x^6-1}{x-1}$
   (b) $\lim_{x\to \infty} \frac{e^{2x}}{x^3}$
   (c) $\lim_{x\to 4} \frac{e^x-e^4}{x-4}$
   (d) $\lim_{x\to 0} \frac{e^{2x}-1-2x}{x^2}$

2. (8 points) Using integration by parts, find $\int xe^{2x}dx$

3. (8 points) Let $f(x) = x^2 + 3x + 2$ on the interval $[-5, 0]$. Identify the critical points and find the maximum value and minimum value on the given interval $[-5, 0]$.

4. (8 points) Make an analysis and then sketch the graph of the function

$$f(x) = (x-2)(x-1)(x+1) = x^3 - 2x^2 - x + 2.$$

5. (8 points) Evaluate the following indefinite integrals
   (a) $\int dx$

   (b) $\int x^{1/4} dx$

   (c) $\int 3e^t dt$

   (d) $\int \frac{z^2}{1+z^3} dz$

6. (8 points) Evaluate the following definite integrals
(a) $\int_0^2 x^3 dx$

(b) $\int_1^2 (4x^3 + 7) dx$

(c) $\int_1^1 \frac{2}{t^3} dt$

(d) $\int_e^{e^2} \frac{lnt}{t} dt$

7. (4 points) Calculate the Riemann sum $\sum_{i=1}^{n} f(c_i)\Delta x_i$ when $f(x) = x$; the partition is $P : 0 < 0.5 < 1.25 < 1.75 < 2.5 < 3$; and sample points are $c_1 = 0, c_2 = 1.25, c_3 = 1.75, c_4 = 2, c_5 = 3$.

8. (8 points) Find the area of the region bounded by $f(x) = x^2 - 2$ and $g(x) = x$.

9. (8 points) Find local extrema of the function

$$f(x, y) = x^2 + y^2 + 2x - 6y + 14$$

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10. (8 points) For $z = f(x, y) = 2x^2 - 3x^2y + 5y + 1$, find

(a) $\frac{\partial z}{\partial x}$

(b) $f_y(x, y)$

(c) $f_x(2, 3)$

11. (8 points) Use method of Lagrange multiplier to solve optimal problem: Maximize $f(x, y) = 25 - x^2 - y^2$ subject to $x + y = 4$.

12. (8 points) Evaluate $\iint_R (2x - y)dA$ over the region $R = \{(x, y)| - 1 \leq x \leq 5, 2 \leq y \leq 4\}$.

Bonus (20 Points)

13. (10 points) An oil field is estimated to produce oil at a rate of $R(t)$ thousand barrels per month t months from now, as given by

$$R(t) = 10te^{-0.1t}$$

. Use an appropriate definite integral to find the total production (to the nearest thousand barrels) in the first year of operation.

14. (10 points) Minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to $2x - y + 3z = -28$.