1 Solution To Homework 7

Solution to Problem 1:
The length of each subinterval is $\Delta x = \frac{5-(-5)}{5} = 2$, so the Riemann sum $S_5 = \sum_{k=1}^{5} f(c_k)\Delta x = 2(f(c_1) + f(c_2) + \cdots + f(c_5)) = -60$

Solution to Problem 2:
(A) $\int_0^3 f(x)dx = \int_0^2 f(x)dx + \int_2^3 f(x)dx = 2 + \frac{10}{3} = \frac{16}{3}$.

(B) $\int_0^2 (2f(x) - 11g(x))dx = 2\int_0^2 f(x)dx - 11\int_0^2 g(x)dx = 2\times 2 - 11\times 3 = -29$

Solution to Problem 3:
(A) $\frac{d}{dx}(\int e^{-x^2}dx) = e^{-x^2}$

(B) $\int \frac{d}{dx}(\sqrt{4+5x})dx = \sqrt{4+5x} + C$

Solution to Problem 4:
(A) $\int_1^1 (x+1)^9 dx = 0$.

(B) $\int_0^9 (4-t^2)dt = (4t - \frac{t^3}{3})|_{t=0}^{t=9} = -207$

(C) $\int_{10}^{20} 5dx = 5x|_{10}^{20} = 50$

(D) $\int_{-1}^{1} \sqrt{1+x}dx = \int_0^2 \sqrt{u}du = \frac{u^{3/2}}{3/2}|_0^2 = \frac{4\sqrt{2}}{3}$

2 Solution To Homework 8

Solution to Problem 1:
(a) $\int_{-2}^{-1} \frac{1}{x}dx = \ln|x||_{-2}^{-1} = \ln|-1| - \ln|-2| = -\ln 2$.

(b) $\int_1^1 e^{x^2}dx = 0$

(c) $\int_2^3 12(x^2-5)^5 xdx = \int_{-1}^{4} 6u^5 du = u^6|_{-1}^{4} = 4^6 - (-1)^6 = 4^6 - 1$

(d) $\int_{-5}^{10} e^{-0.05x}dx = \int_{0.25}^{-0.5} -20e^u du = -20e^u|_{0.25}^{-0.5} = 20(e^{0.25} - e^{-0.5})$

(e) $\int_2^5 \frac{1}{\sqrt{6-t}}dt = \int_4^1 \frac{1}{\sqrt{u}}(-du) = \int_1^4 u^{1/2}du = \frac{u^{3/2}}{3/2}|_1^4 = \frac{14}{3}$

(f) $\int_3^9 \frac{1}{x-1}dx = \int_2^8 \frac{1}{u}du = \ln|u||_2^8 = \ln 4$

Solution to Problem 2
(a) $\int xe^{2x}dx = \int xd(\frac{1}{2}e^{2x}) = x\times(\frac{1}{2}e^{2x}) - \int \frac{1}{2}e^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$

(b) $\int x\ln 2xdx = \int \ln 2xd(\frac{1}{2}x^2) = \frac{1}{2}x^2\ln 2x - \int \frac{1}{2}x^2 \times \frac{1}{x}dx = \frac{1}{2}x^2\ln 2x - \frac{1}{4}x^2$.

Solution to Problem 3 (a) The area is

$$\int_{-1}^{3} (x^2+1)dx = (\frac{x^3}{3} + x)|_{-1}^{3} = \frac{40}{3}$$

(b) The area is

$$\int_{-2}^{3} \frac{1}{2}x + 3 - (-x^2+1)dx = \int_{-2}^{3} x^2 + \frac{1}{2}x + 2dx = (\frac{x^3}{3} + \frac{x^2}{4} + 2x)|_{-2}^{1} = \frac{33}{4}$$

(c) The area is

$$\int_{-2}^{2} (-x+10)dx = (-\frac{x^2}{2} + 10x)|_{-2}^{2} = 40$$

(d) The area is

$$\int_1^e (0-(-\frac{1}{x}))dx = \int_1^e \frac{1}{x}dx = (\ln|x|)|_1^e = 1$$

3 Solution to Homework 9

Solution to Problem 1
$f(-2,2,3) = 2(-2)^2 - 3(-2)(2) + 3(3) + 1 = 30$

Solution to Problem 2 (A) $\frac{f(x+h,y)-f(x,y)}{h} = \frac{2hx+h^2}{h} = 2x+h$

(B) $\frac{f(x,y+k)-f(x,y)}{k} = \frac{4ky+2k^2}{k} = 4y+2k$

Solution to Problem 3
Matched Problem 6, Section 8.1:
$A(0,0,0)$, $B(2,0,0)$, $C(2,4,0)$, $D(0,4,0)$, $E(0,0,3)$, $F(2,0,3)$, $G(2,4,3)$, $H(0,4,3)$
Matched Problem 7(A), Section 8.1:
This one is not required in the final exam.

Solution to Problem 4
Matched Problem 1, Section 8.2:
(A) $\frac{\partial z}{\partial x} = 4x - 6xy$
(B) $f_x(2,3) = 4(2) - 6(2)(3) = 8 - 36 = -28$
Matched Problem 2, Section 8.2:
(A) $\frac{\partial z}{\partial y} = 5(x^2+2xy)^4(2x) = 10x(x^2+2xy)^4$
(B) $f_x(x,y) = 5(x^2+2xy)^4(2x+2y)$, so $f_x(1,0) = 5(1^2+0)^4(2*1+2*0) = 10$
Matched Problem 5, Section 8.2:
(A) $\frac{\partial z}{\partial x} = 3x^2y$, so $\frac{\partial^2 z}{\partial y \partial x} = 3x^2$
(B) $\frac{\partial z}{\partial y} = x^3 - 8y^3$, so $\frac{\partial^2 z}{\partial y^2} = -24y^2$
(C) $f_{xy}(2,3) = 3(2)^2 = 12$
(D) $f_{yx}(2,3) = 3(2)^2 = 12$

Solution to Problem 5
Matched Problem 1, Section 8.3
Step 1. Find critical points:
$f_x(x,y) = 2x - 10 = 0$
$f_y(x,y) = 2y - 2 = 0$
So the only critical point is $(a,b) = (5,1)$.
Step 2. Compute $A = f_{xx}(5,1), B = f_{xy}(5,1), C = f_{yy}(5,1)$:
$f_{xx}(x,y) = 2$, so $A = 2$
$f_{xy}(x,y) = 0$, so $B = 0$
$f_{yy}(x,y) = 2$, so $C = 2$
Step 3. Evaluate $AC - B^2$ and clssify the critical point $(5,1)$:
$AC - B^2 = 4 > 0$, $A = 2 > 0$ and hence by Theorem 2, Section 8.3, $f(5,1) = 10$ is the local minimum.

4 Solution to Homework 10

Solution to Problem 1
Step 1. Find critical points of $f(x, y) = x^3 + y^2 - 6xy$.
Solve for the folowing two equations

$$f_x(x, y) = 3x^2 - 6y = 0$$

$$f_y(x, y) = 2y - 6x = 0$$

and find that $(0, 0)$ and $(6, 18)$ are two critical points.
Test $(0, 0)$: Step 2. Compute $A = f_{xx}(0, 0), B = f_{xy}(0, 0), C = f_{yy}(0, 0)$.
$f_{xx}(x, y) = 6x$; so, $A = f_{xx}(0, 0) = 0$
$f_{xy}(x, y) = -6$; so, $B = f_{xy}(0, 0) = -6$
$f_{yy}(x, y) = 2$; so, $C = f_{yy}(0, 0) = 2$
Step 3. Evaluate $AC - B^2$ and classify the critical point $(0, 0)$
$AC - B^2 = -36 < 0$
Therefore, case 3 in Theorem 2, section 8.3 implies that f has a saddle point at $(0, 0)$.
Test $(6, 18)$: Step 2. Compute $A = f_{xx}(6, 18), B = f_{xy}(6, 18), C = f_{yy}(6, 18)$.
$A = f_{xx}(6, 18) = 6(6) = 36$
$B = f_{xy}(6, 18) = -6$
$C = f_{yy}(6, 18) = 2$
Step 3. Evaluate $AC - B^2$ and classify the critical point $(6, 18)$
$AC - B^2 = 36 > 0$, $A > 0$
Therefore, Theorem 2, section 8.3 implies that $f(6, 18) = -108$ is a local minimum of f.
Solution to Problem 2
Matched Problem 1, Section 8.6
(A) $\int(4xy + 12x^2y^3)dy = 2xy^2 + 3x^2y^4 + C(x)$, where $C(x)$ is an arbitrary function of x.
(B) $\int(4xy + 12x^2y^3)dx = 2yx^2 + 4x^3y^3 + C(y)$, where $C(y)$ is an arbitrary function of y.
Matched Problem 2, Section 8.6
(A) $\int_0^1(4xy + 12x^2y^3)dy = (2xy^2 + 3x^2y^4)|_{y=0}^{y=1} = 2x + 3x^2$
(B) $\int_0^3(4xy + 12x^2y^3)dx = (2yx^2 + 4x^3y^3)|_{x=0}^{x=3} = 18y + 108y^3$
Matched Problem 3, Section 8.6
(A) $\int_0^3[\int_0^1(4xy + 12x^2y^3)dy]dx = \int_0^3(2x + 3x^2)dx = (x^2 + x^3)|_{x=0}^{x=3} = 36$
(B) $\int_0^1[\int_0^3(4xy+12x^2y^3)dx]dy = \int_0^1(18y+108y^3)dy = (9y^2+27y^4)|_{y=0}^{y=1} = 36$

Solution to Problem 3 Matched Problem 4, Section 8.6

$$\begin{aligned}
\iint_R (2x-y)dA &= \int_0^1 [\int_{-1}^1 (2x-y)dy]dx \\
&= \int_0^1 (2xy - \frac{y^2}{2})|_{y=-1}^{y=1} dx \\
&= \int_0^1 4x dx \\
&= 2x^2|_{x=0}^{x=1} \\
&= 2
\end{aligned}$$

Another way to evaluate this double integral is

$$\begin{aligned}
\iint_R (2x-y)dA &= \int_{-1}^1 [\int_0^1 (2x-y)dx]dy \\
&= \int_{-1}^1 (x^2 - yx)|_{x=0}^{x=1} dy \\
&= \int_{-1}^1 (1-y)dy \\
&= (y - \frac{y^2}{2})|_{y=-1}^{y=1} \\
&= 2
\end{aligned}$$

Matched Problem 5, Section 8.6

$$\begin{aligned}
\iint_R \frac{x}{y^2} e^{\frac{x}{y}} dA &= \int_0^1 [\int_1^2 \frac{x}{y^2} e^{\frac{x}{y}} dy]dx \\
&= \int_0^1 -e^{\frac{x}{y}}|_{y=1}^{y=2} dx \\
&= \int_0^1 (e^x - e^{\frac{x}{2}})dx \\
&= (e^x - \frac{1}{2} e^{\frac{x}{2}})|_{x=0}^{x=1} \\
&= e - \frac{1}{2} e^{\frac{1}{2}} - \frac{1}{2}
\end{aligned}$$