1 Solution To Homework 6

Solution to problem 1:
(A)

$$\begin{aligned}\int \frac{e^x - 3x}{4} dx = & \int (\frac{e^x}{4} - \frac{3x}{4}) dx \\ = & \frac{1}{4}\int e^x dx - \frac{3}{4}\int x dx \\ = & \frac{1}{4}e^x - \frac{3}{4} \times \frac{x^2}{2} + C \\ = & \frac{1}{4}e^x - \frac{3}{8}x^2 + C\end{aligned}$$

(B) Since $\frac{1}{\sqrt{u}} = u^{-1/2}$, $\int \frac{du}{\sqrt{u}} = \int u^{-1/2} du = 2u^{1/2} + C$.
(C)

$$\begin{aligned}\int \frac{1 - 3x^4}{x^2} dx = & \int (\frac{1}{x^2} - 3x^2) dx \\ = & \int x^{-2} dx - 3\int x^2 dx \\ = & -\frac{1}{x} - x^3 + C\end{aligned}$$

(D) $\int \frac{6dm}{m^2} = 6\int m^{-2} dm = -6m^{-1} + C = -\frac{6}{m} + C$.

Solution to problem 2:
(A) The general solution for the differential equation $C'(x) = 6x^2 - 4x$ is

$$\begin{aligned}C(x) = & \int C'(x) dx \\ = & \int (6x^2 - 4x) dx \\ = & 6\int x^2 dx - 4\int x dx \\ = & 6 \times \frac{x^3}{3} - 4 \times \frac{x^2}{2} + C \\ = & 2x^3 - 2x^2 + C\end{aligned}$$

, where the constant C is specified by the initial condition $C(0) = 3000$: when $x = 0$, $C(0) = 2 \times 0^3 - 2 \times 0^2 + C = 3000$ and hence $C = 3000$. In summary, $C(x) = 2x^3 - 2x^2 + 3000$.
(B) The general solution for the differential equation $\frac{dx}{dt} = 4e^t - 2$ is

$$\begin{aligned}x(t) = & \int (4e^t - 2) dt \\ = & 4\int e^t dt - 2\int dt \\ = & 4e^t - 2t + C\end{aligned}$$

, where the constant C is specified by the initial condition $x(0) = 1$ as follows. $x(0) = 4 \times e^0 - 2 \times 0 + C = 1$, which implies that $C = 1 - 4 = -3$. In summary, $x(t) = 4e^t - 2t - 3$.

Solution to problem 3:

(A) Let $u = 2x^3 - 3$, then $du = 6x^2 dx$. By substitution,

$$\begin{aligned} \int (2x^3-3)^4(6x^2)dx &= \int u^4 du \\ &= \frac{u^5}{5} + C \\ &= \frac{(2x^3-3)^5}{5} + C. \end{aligned}$$

(B) Let $u = x^2 - 9$, then $du = 2xdx$, $xdx = \frac{1}{2}du$. By substitution,

$$\begin{aligned} \int \frac{x}{x^2-9}dx &= \int \frac{1/2du}{u} \\ &= 1/2\int \frac{du}{u} \\ &= \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln|x^2-9| + C. \end{aligned}$$

(C) Let $u = t^3 + 4$, then $du = 3t^2dt$, $t^2dt = \frac{1}{3}du$. By substitution,

$$\begin{aligned} \int 5t^2(t^3+4)^{-2}dt &= \int 5u^{-2} \times \frac{1}{3}du \\ &= \frac{5}{3}\int u^{-2}du \\ &= \frac{5}{3} \times \frac{u^{-1}}{-1} + C \\ &= -\frac{5}{3(t^3+4)} + C \end{aligned}$$

(D) Let $u = x + 1$, then $du = dx$. By substitution,

$$\begin{aligned} \int x\sqrt{x+1}dx &= \int (u-1)\sqrt{u}du \\ &= \int (u^{3/2} - u^{1/2})du \\ &= \int u^{3/2}du - \int u^{1/2}du \\ &= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C \\ &= \frac{2(x+1)^{5/2}}{5} - \frac{2(x+1)^{3/2}}{3} + C. \end{aligned}$$

(E) Let $u = 1 - x$, then $du = -dx$. By substitution,

$$\begin{aligned}\int e^{1-x} dx = & \int e^u(-du) \\ = & -\int e^u du \\ = & -e^u + C \\ = & -e^{1-x} + C.\end{aligned}$$

(F) Let $u = \ln x$, then $du = \frac{dx}{x}$. By substitution,

$$\begin{aligned}\int \frac{(\ln x)^3}{x} dx = & \int u^3 du \\ = & \frac{u^4}{4} + C \\ = & \frac{(\ln x)^4}{4} + C.\end{aligned}$$

Solution to problem 4:
(A) Since the partition is obtained by dividing the interval $[0, 3]$ into three subintervals of equal length, the length of each subinterval Δx is $\Delta x = \frac{3-0}{3} = 1$. Endpoints of each subintervals are: $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.

Therefore the partition is $P_3 : x_0 = 0 < x_1 = 1 < x_2 = 2 < x_3 = 3$. The Riemann sum in this case is:

$$\begin{aligned}S_3 = & f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x \\ = & f(0.7) \times 1 + f(1.8) \times 1 + f(2.4) \times 1 \\ = & 0.49 + 3.24 + 5.76 \\ = & 9.49\end{aligned}$$

(B) In this question (B), the partition is the same as in (A), which is $P_3 : x_0 = 0 < x_1 = 1 < x_2 = 2 < x_3 = 3$. Now the smaple points are the right endpoints of each subintervals. In other words, those sample points are $c_1 = x_1 = 1, c_2 = x_2 = 2, c_3 = x_3 = 3$.

So the Riemann sum in this case is:

$$\begin{aligned}S_3 = & f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x \\ = & f(1) \times 1 + f(2) \times 1 + f(3) \times 1 \\ = & 14\end{aligned}$$