1 Solution To Homework 5

Solution to problem 1(A):
Step 1: Find Critical Values
The function $f(x)$ is a polynomial, so we only need to consider the first type critical values: those numbers x in the domain satisfying $f'(x) = 0$.

$$f'(x) = 3x^2 - 12$$

Solve the equation $f'(x) = 0$ for x and find that $x = -2$ or $x = 2$. The domain is $[-5, 5]$ and hence $x = -2, 2$ both are in the domain, which implies that $x = 2$ and $x = -2$ are critical values for $f(x)$ in the domain $[-5, 5]$.

Step 2: Evaluate $f(x)$ at endpoints and criticals values

x	$f(x)$
-5	$f(-5) = -65$
-2	$f(-2) = 16$
2	$f(2) = -16$
5	$f(5) = 65$

Step 3: Compare function values in Step 2, find largest and smallest ones among all
From Step 2, we can see that the function $f(x)$ obtains the absolute maximum value 65 at $x = 5$, absolute minimum value -65 at $x = -6$.

Solution to problem 1(B):
Step 1: Find Critical Values
From Step 1. in (A), we know that when $x = -2$ or 2, $f'(x) = 0$. Since $-2, 2$ are in $[-3, 3]$, they are critical values for the function $f(x)$ with the domain $[-3, 3]$.

Step 2: Evaluate $f(x)$ at endpoints and criticals values

x	$f(x)$
-3	$f(-3) = -9$
-2	$f(-2) = 16$
2	$f(2) = -16$
3	$f(3) = 9$

Step 3: Compare function values in Step 2, find largest and smallest ones among all
From Step 2, we can see that the function $f(x)$ obtains the absolute maximum value 16 at $x = -2$, absolute minimum value -16 at $x = 2$.

Solution to problem 1(C):
Step 1: Find Critical Values
From Step 1. in (A), we know that when $x = -2$ or 2, $f'(x) = 0$. Since -2 is in $[-3, 1]$ and 2 is not, $x = -2$ is the only critical value for the function $f(x)$ with the domain $[-3, 1]$.

Step 2: Evaluate $f(x)$ at endpoints and criticals values

x	$f(x)$
-3	$f(-3) = -9$
-2	$f(-2) = 16$
1	$f(1) = -11$

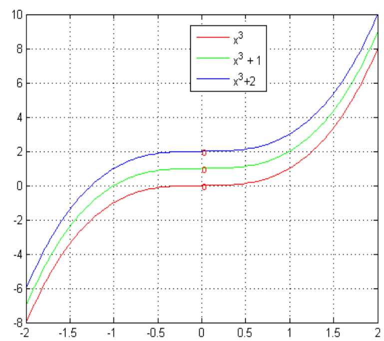
Step 3: Compare function values in Step 2, find largest and smallest ones among all

From Step 2, we can see that the function $f(x)$ obtains the absolute maximum value 16 at $x = -2$, absolute minimum value -11 at $x = 1$. This problem is similar to Problem 2 in Homework 4, so we don't repeat the steps here.

Solution to problem 3:

(A) All antiderivatives of $f(x) = 3x^2$ are $x^3 + C$, where C is an arbitrary constant.

(B) See the figure 1.



(C) The graphs of the three antiderivatives are the vertical shift of each other.

Solution to problem 4:

(A) $\int 3dx = 3x + C$

(B) $\int 10e^t dt = 10 \int e^t dt = 10e^t + C$

(C) $\int 3x^4 dx = 3 \int x^4 dx = 3 \times \frac{x^5}{5} + C = \frac{3}{5}x^5 + C$

(D) By linearity, $\int (2x^5 - 3x^2 + 2)dx = 2 \int x^5 dx - 3 \int x^2 dx + 2 \int 1dx = \frac{2}{6}x^6 - x^3 + 2x + C$

(E) By lineary, $\int (\frac{3}{x} - 4e^x)dx = 3 \int \frac{1}{x} dx - 4 \int e^x dx = 3 \ln |x| - 4e^x + C$

(F) $\int \frac{x^4 - 8x^3}{x^2} dx = \int (x^2 - 8x)dx = \int x^2 dx - 8 \int x dx = \frac{x^3}{3} - 4x^2 + C$

(G) Since $(x^2 - 1)(x + 3) = x^3 + 3x^2 - x - 3$, $\int (x^2 - 1)(x + 3)dx = \int (x^3 + 3x^2 - x - 3)dx = \frac{x^4}{4} + x^3 - \frac{x^2}{2} - 3x + C$.

(H) Since $\sqrt{x} = x^{\frac{1}{2}}$ and $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$, it follows that $\int (8\sqrt{x} - \frac{6}{\sqrt{x}})dx = 8 \int x^{\frac{1}{2}} dx - 6 \int x^{-\frac{1}{2}} dx = 8\frac{x^{3/2}}{3/2} - 6\frac{x^{1/2}}{1/2} + C = \frac{16}{3}x^{3/2} - 12x^{1/2} + C$