# 1 Solution To Homework 4

**Solution to problem 1:**
*Step a.* Analyze $f(x)$.

The domain of the given function is its natural domain $(-\infty, \infty)$.

The $y$ intercept is $f(0) = 10$.

*Step b.* Analyze $f'(x)$.

$$f'(x) = 4x^3 - 12x^2$$

Solve the equation $f'(x) = 0$ for $x$, and obtain two solutions $x = 0$ or $x = 3$, which are partition numbers for $f'(x)$.

By performing first derivative test, we establish the following first derivative test:

| | | | |
|---|---|---|---|
| intervals | $(-\infty, 0)$ | $(0, 3)$ | $(3, \infty)$ |
| test numbers $x$ | $-1$ | $1$ | $4$ |
| $f'(x)$ | $-$ | $-$ | $+$ |
| behavior of $f(x)$ | decreasing | decreasing | increasing |

By the first derivative test, we know that the function $f(x)$ has a local minimum $f(3) = -17$ at $x = 3$, is decreasing on intervals $(-\infty, 0)$ and $(0, 3)$, and increasing on the interval $(3, \infty)$.

*Step c.* Analyze $f''(x)$.

$$f''(x) = 12x^2 - 24x$$

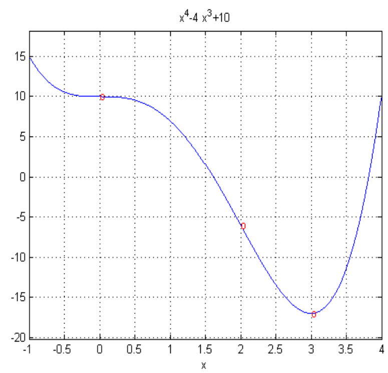
Solve the equation $f''(x) = 0$ for $x$, and obtain two solutions $x = 0$ or $x = 2$, which are partition numbers for $f''(x)$.

By performing second derivative test, we establish the following second derivative test:

| | | | |
|---|---|---|---|
| intervals | $(-\infty, 0)$ | $(0, 2)$ | $(2, \infty)$ |
| test numbers $x$ | $-1$ | $1$ | $3$ |
| $f''(x)$ | $+$ | $-$ | $+$ |
| behavior of $f(x)$ | concave up | concave down | concave up |

By the second derivative test, we know that the function $f(x)$ has two inflection points: $(0, 10)$, $(2, -6)$. It is concave up on intervals $(-\infty, 0)$ and $(2, \infty)$, concave down on the interval $(0, 2)$.

*Step d.* Sketch the graph for f. See the figure 1.




**Solution to Problem 2:**

*Step 1.* Analyze $f(x)$

The domain of the function $f(x)$ is $(-\infty, 1) \cup (1, \infty)$.

Intercepts: x-intercept and y-intercept occur at $(0, 0)$.

Since

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x}{1-x} = -2$$

and

$$\lim_{x\to 1^+} f(x) = \lim_{x\to\infty} \frac{2x}{1-x} = -\infty,$$

$f(x)$ has two asymptotes, horizontal asymptote $y = -2$ and vertical asymptote $x = 1$.

*Step 2.* Analyze $f'(x)$

$$f'(x) = \frac{2}{(1-x)^2}.$$

Solve $f'(x) = 0$ for $x$ and find no solution, which means there is no first type of partition number for $f'(x)$. When $x = 1$, the derivative $f'(x)$ does not exist, which implies that $x = 1$ is the second type of partition number for $f'(x)$.

Next, we perform the first derivative test for $f(x)$.

| | | |
|---|---|---|
| intervals | $(-\infty, 1)$ | $(1, \infty)$ |
| test numbers $x$ | 0 | 2 |
| $f'(x)$ | + | + |
| behavior of $f(x)$ | increasing | increasing |

By the first derivative test, we know that the function $f(x)$ is increasing on the interval $(-\infty, 1)$ and $(1, \infty)$.

*Step 3.* Analyze $f"(x)$.

$$f''(x) = \frac{4}{(1-x)^3}.$$

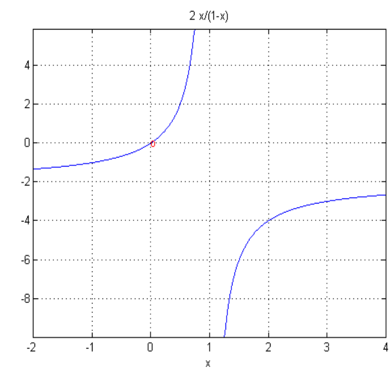
It is easy to see that partition numbers for $f''(x)$ is $x = 1$.

Next, we perform the second derivative test for $f(x)$.

| intervals | $(-\infty, 1)$ | $(1, \infty)$ |
|---|---|---|
| test numbers $x$ | 0 | 2 |
| $f''(x)$ | $-$ | $+$ |
| behavior of $f(x)$ | concave down | concave up |

By the second derivative test, we know that the function $f(x)$ is concave up on the interval $(-\infty, 1)$ and concave down on the interval $(1, \infty)$.

*Step 4.* Sketch the graph of $f(x)$ See the figure 2.




**Solution to Problem 3:**

(a) By LH Rule, we have $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-1/2}}{1} = \frac{\frac{1}{2}\times 1^{-1/2}}{1} = \frac{1}{2}$

(b) By LH Rule, we have $\lim_{x\to 4} \frac{e^x - e^4}{x-4} = \lim_{x\to 4} \frac{e^x}{1} = e^4$

(c) By LH Rule, we have $\lim_{x\to 1} \frac{\ln x}{(x-1)^3} = \lim_{x\to 1} \frac{1/x}{3(x-1)^2} = \infty$.

(d) By LH Rule, we have $\lim_{x\to 0} \frac{e^{2x}-1-2x}{x^2} = \lim_{x\to 0} \frac{2e^{2x}-2}{2x} = \lim_{x\to 0} \frac{2e^{2x}}{1} = 2$.

(e) By LH Rule, we have $\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$.