1 Solution To Homework 3

Solution to Problem 1(A):
Step 1. Find domain.

The domain of the given function is its natural domain $(-\infty, \infty)$.

Step 2. Find partition numbers for $f(x)$.

$$f'(x) = e^{-x}$$
$$f"(x) = -e^{-x}$$

We can conclude that $f''(x)$ is well-defined and nonzero for all x in the domain $(-\infty, \infty)$, and hence there is no partition number for $f(x)$. There is only one interval $(-\infty, \infty)$. In other words, the function $f''(x)$ keeps the same sign in the domain $(-\infty, \infty)$.

Step 3. Second derivative test.

By perform second derivative test, we establish the following second derivative test:

Intervals	$(-\infty, \infty)$
x	0
$f"(x)$	$-e^0 = -1 < 0$
Concavity of $f(x)$	concave down

Step 4. Find Concavity.

From the Step 3., we can conclude that the function $f(x)$ is concave down in the domain $(-\infty, \infty)$.

Solution to Problem 1(B):
Step 1. Find domain.

The domain of the given function is its natural domain $(0, \infty)$.

Step 2. Find partition numbers for $g(x)$.

$$g'(x) = -\frac{1}{x}$$
$$g"(x) = \frac{1}{x^2}$$

We can conclude that $g''(x)$ is well-defined and positive for all x in the domain $(0, \infty)$, and hence there is no partition number for $g(x)$. There is only one interval $(0, \infty)$that second derivative test to be performed on. In other words, the function $g''(x)$ keeps the same sign in the domain $(0, \infty)$.

Step 3. Second derivative test.

By perform second derivative test, we establish the following second derivative test:

Intervals	$(0, \infty)$
Test numbers x	1
$g"(x)$	$\frac{1}{1^2} = 1 > 0$
Concavity of $g(x)$	concave up

Step 4. Find Concavity.

From *Step 3.*, we can conclude that the function $g(x)$ is concave up in the domain $(0, \infty)$.

Solution to Problem 1(C):

Step 1. Find domain.

The domain of the given function $h(x)$ is its natural domain $(-\infty, \infty)$.

Step 2. Find partition numbers for $h(x)$.

$$h'(x) = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$h''(x) = \frac{1}{3} \times \frac{-2}{3}x^{-2/3-1} = \frac{-2}{9x^{5/3}}$$

We can conclude that $h"(x)$ has a partition number $x = 0$, which is of second type $h"(x)$ DNE.

Step 3. Second derivative test.

By perform second derivative test, we establish the following table:

Intervals	$(-\infty, 0)$	$(0, \infty)$
x	-1	1
$h"(x)$	$\frac{-2}{9\times(-1)^{5/3}} > 0$	$\frac{-2}{9\times(1)^{5/3}} < 0$
Concavity of $f(x)$	concave up	concave down

Step 4. Find Concavity.

From the Step 3., we can conclude that the function $h(x)$ is concave up in the interval $(-\infty, 0)$ and concave down in the interval $(0, \infty)$.

Solution to Problem 2:

Step 1. Find the second derivative $f''(x)$.

$$f'(x) = 12x^2 + 42x + 36$$

$$f''(x) = 24x + 42$$

Step 2. Find partition numbers and corresponding intervals.

Solving the equation $24x + 42 = 0$, we obtain that $x = -7/4$, which is the partition number for $f''(x)$. It partitions the domain $(-\infty, \infty)$ into two intervals $(-\infty, -7/4)$ and $(-7/4, \infty)$.

Step 3. Second derivative test.

Intervals	$(-\infty, -7/4)$	$(-7/4, \infty)$
x	-2	0
$f''(x)$	$-$	$+$
Concavity of $f(x)$	concave down	concave up

Step 4. Concavity

From *Step 3.*, we can conclude that the function $f(x)$ is concave down in the interval $(-\infty, -7/4)$ and concave up in the interval $(-7/4, \infty)$.

Solution to Problem 3:

Step 1. Find the second derivative $f''(x)$.

$$f'(x) = 3x^2 - 12x + 9$$
$$f''(x) = 6x - 12$$

Step 2. Find partition numbers and corresponding intervals.
Solving the equation $6x - 12 = 0$, we obtain that $x = 2$, which is the partition number for $f''(x)$. It partitions the domain $(-\infty, \infty)$ into two intervals $(-\infty, 2)$ and $(2, \infty)$.

Step 3. Second derivative test.

Intervals	$(-\infty, 2)$	$(2, \infty)$
x	0	3
$f''(x)$	$-$	$+$
Concavity of $f(x)$	concave down	concave up

Step 4. Inflection points
From *Step 3.*, we can conclude that the function $f(x)$ changes concavity across $x = 2$ and hence $(2, f(2)) = (2, 3)$ is the inflection point.

Solution to Problem 4:

Step 1. Find the critical values of $f(x)$.

$$f'(x) = 3x^2 - 12$$

Solving the equation $3x^2 - 12 = 0$, we obtain $x = -2$ or $x = 2$, which are critical values.

Step 2. Evaluate the second derivative $f''(x)$ at critical values and perform second derivative test for local extrema.

$$f''(x) = 6x.$$

At $x = -2$, $f''(-2) = 6 * (-2) < 0$, which implies that the function $f(x)$ has a local maxima at $x = -2$. At $x = 2$, $f''(2) = 6 * 2 > 0$, which implies that the function $f(x)$ has a local minima at $x = 2$.