1 Solution To Homework 2

Solution to Problem 1:
Step 1. Find the domain.
The domain of the function $f(x)$ is not specified, so the domain here is the natural domain, which is $(0, \infty)$.
Step 2. Find the derivative $f^{'}(x)$.

$$f^{'}(x) = \frac{8}{x} - 2x. \tag{1}$$

Step 3. Find the partition numbers.
Type 1: $f^{`}(x) = 0$. From the equation (1), we have that

$$\frac{8}{x} - 2x = 0. \tag{2}$$

Solve the equation ((2) and obtain that $x = -2$, or $x = 2$. Since -2 is not in the domain $(0, \infty)$ and does not partition the domain into intervals, $x = -2$ is not a partition number. The onlye type 1 partition number is -2. Type 2: $f^{`}(x)$ DNE. When $x = 0$, $f^{`}(x)$ DNE. However, 0 does not partition the domain $(0, \infty)$ into intervals, which means 0 is not a partition number.

Overall, the only partition number is $x = 2$.

Step 4. Find Critical Values of $f(x)$.

$x = 2$ is the only partition number and it is in the domain $(0, \infty)$. Therefore, $x = 2$ is the only critical value of $f(x)$.

Solution to Problem 2(a):
Step 1. Find the derivative of $f^{`}$.

$$f^{`}(x) = 3x^2 - 18x + 24.$$

Step 2. Find the critical values of $f(x)$.
For a polynomial, the possible critical values should be Type I: $f^{`}(x) = 0$. Therefore, we solve the following equation for x:

$$3x^2 - 18x + 24 = 3(x - 2)(x - 4) = 0,$$

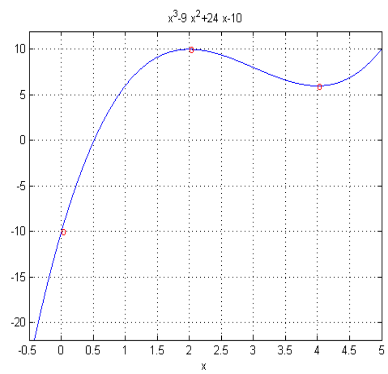
and find solutions are $x = 2$ or $x = 4$, which are critical values for the function $f(x)$. Meanwhile, those two critical values $x = 2$ or $x = 4$ are also partition numbers.

Solution to Problem 2(b):
Step 1. Perform first derivative test.

Intervals	$(-\infty, 2)$	$(2, 4)$	$(4, \infty)$
test numbers x	0	3	5
$f'(x)$	$+$	$-$	$+$
$f(x)$	increasing	decreasing	increasing

Step 2. Find the local maxima and minima.
The first derivative test above tells us the function $f(x)$ has a local maxima $f(2) = 10$ at $x = 2$, and a local minima $f(4) = 6$ at $x = 4$.

Step 3. Sketch the graph of $f(x)$.



Solution to Problem 2(b):
Step 1. Find the derivative of the function.

$$y^{'} = F'S + FS' = 4x\sqrt{x} + (2x^2 - 4)\frac{1}{2}x^{-1/2} = 4x\sqrt{x} + (x^2 - 2)x^{-1/2}. \quad (3)$$

Step 2. Find the corresponding differential.

$$dy = (4x\sqrt{x} + (x^2 - 2)x^{-1/2})dx. \quad (4)$$

Solution to Problem 3(a):
Step 1. Find the critical values of $f(x)$.

Since $f'(x) = 3/2$ for all x in the domain $[-2, 3]$, there is no critical value in the domain.

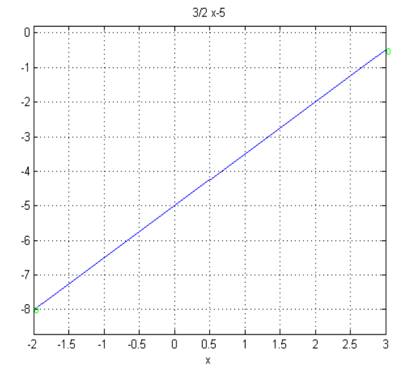
Step 2. Evaluate the function $f(x)$ at critical values and endpoints.

From Step 1, we know that the function $f(x)$ has no critical value in the domain, so we only need to evaluate the function $f(x)$ at endpoints, which are: $f(-2) = \frac{3}{2} * (-2) - 5 = -8$ and $f(3) = \frac{3}{2} * 3 - 5 = -\frac{1}{2}$.

Step 3. Compare function values in Step 2. and find absolute max and min.

By compare the function values $-\frac{1}{2}$ and -8, we conclude that the absolute maximum value of $f(x)$ is $-\frac{1}{2}$, and the absolute minimum value of $f(x)$ is -8.

Solution to Problem 3(b):



Solution to Problem 4(a):

Step 1. Find the critical values of $f(x)$.

Since $f'(x) = \frac{2}{x^3} > 0$ for all x in the domain $[0.5, 2]$, there is no critical value in the domain.

Step 2. Evaluate the function $f(x)$ at critical values and endpoints.

From Step 1, we know that the function $f(x)$ has no critical value in the domain, so we only need to evaluate the function $f(x)$ at endpoints, which are: $f(0.5) = -\frac{1}{0.5^2} = -4$ and $f(2) = -\frac{1}{2^2} = -0.25$.

Step 3. Compare function values in Step 2. and find absolute max and min.

By compare the function values -0.25 and -4, we conclude that the absolute maximum value of $f(x)$ is -0.25, and the absolute minimum value of $f(x)$ is -4.

Solution to Problem 4(b): See the figure