1 Solution To Homework 1

Solution to Problem 1:
Step 1. Find the derivative of $f(x)$.

$$f^{'}(x) = 1/3(1+x)^{1/3-1} * (1) = \frac{1}{3(1+x)^{2/3}}. \tag{1}$$

Step 2. Evaluate the derivative at $x = 0$, which is the slope of the corresponding tangent line.

$$f^{'}(0) = \frac{1}{3(1+0)^{2/3}} = \frac{1}{3}. \tag{2}$$

Step 3. Find the linear approximation as required.

$$L(x) = f(0) + f^{'}(0)(x-0) = 1 + \frac{1}{3}x. \tag{3}$$

Solution to Problem 2(a):
Step 1. Find the derivative of the function.

$$(\frac{x}{x+2})^{'} = \frac{T'B - TB'}{B^2} = \frac{2}{(x+2)^2}. \tag{4}$$

Step 2. Find the corresponding differential.

$$d(\frac{x}{x+2}) = \frac{2}{(x+2)^2}dx. \tag{5}$$

Solution to Problem 2(b):
Step 1. Find the derivative of the function.

$$y^{'} = F'S + FS' = 4x\sqrt{x} + (2x^2-4)\frac{1}{2}x^{-1/2} = 4x\sqrt{x} + (x^2-2)x^{-1/2}. \tag{6}$$

Step 2. Find the corresponding differential.

$$dy = (4x\sqrt{x} + (x^2-2)x^{-1/2})dx. \tag{7}$$

Solution to Problem 3:

Step 1. Find the value of x correspond to horizontal tangent line.

$$f^{'}(x) = 2x - 6 = 0, \tag{8}$$

hence $x = 3$, which is the value of x correspond to horiztonal tangent line.

Step 2. Perform first derivative test to decide where $f(x)$ is increasing or decreasing.

From *Step 1.* and the fact that, for a polynomial, the derivative is well-defined for any real number, we know that $x = 3$ is the only partition number. So we have the following first derivative test:

Intervals	$(-\infty, 3)$	$(3, \infty)$
Test numbers x	0	4
$f^{'}(x)$	$2 * 0 - 6 < 0$	$2 * 4 - 6 > 0$
Sign of $f^{'}$	$-$	$+$
Behavior of f	Decreasing	Increasing

Solution to Problem 4:

Step 1. Find the partition numbers.

There are two types of partition numbers.

Step 1.1 The first type satisfy $f^{'}(x) = 0$:

$$f^{'}(x) = -\frac{1}{(x-1)^2} = 0, \tag{9}$$

which has no solution, since the numerator of the fraction in the left side of the equation is nonzero. Therefore there is no first type of partition number.

Step 1.2 The second type satisfy $f^{'}(x)$ DNE (Does Not Exist):

when $x = 1$ which makes $(x-1)^2 = 0$, $f^{'}(1)$ is not well-defined and hence $x = 1$ is the second type of partition number.

Overall, there is only one partition number $x = 1$ for the function $f(x)$ here.

Step 2. Find Critical Values.

Critical values are those partition numbers which lies in the domain of the function $f(x)$. In our case, the domain of the function $f(x)$ is all real numbers except 1. This tells us that the only partition number $x = 1$ is not in the domain, therefore there s no critical value for the function $f(x) = \frac{1}{x-1}$.