Practical Mathematical Handwriting

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Abstract

In mathematical texts and papers (by TEX, of course), there involves different kinds of fonts. For example,

A $\mathbb{A}$ $\mathfrak{A}$ $\mathcal{A}$ $\mathscr{A}$

It looks beautiful under the condition of being used suitably, but writes difficult without any knowledge of calligraphy. In this article, I want to give some practicing handwriting techniques, so that, I believe, it helps you writing them in more beautiful way.

1 Introduction

Firstly, most of the works of calligraphy are written in "slanted" pen. Rather than the ball-pen or modern pen whose head is •, it is more like a brush or nite writer pen whose head is a long rectangle ▮.

Figure 1: square-cut nib, brush pen

When they are writing, the head of the pen is slanted mostly as ／. If the letters lean forward, the horizon line is slim and vertical

line is thick. For example,  . So, it is impossible (non-efficient, more exactly) to use modern pens to write them, but what I want to introduce in this article is to write them more beautiful with normal pens or chalks.

Secondly, clearly, there are a lot of fonts in calligraphy. But I just want to mention some sorts of them appearing in TEX. The brunches of the followings are large topics of history of calligraphy, which are not efficient to discuss here.

- **Romans**, which is one of the most popular font all over the world, is characterized by its serif. The serif is the small line attached to the end of a stroke in a letter. For example,  . In TEX, it is the default font in text and the command is `\textrm` or `\mathrm`.

- **Italic**, thought as a invention of Italian, has its letters lean forward. In TEX, it is the default font in mathematical model, whose command is `\textit` or `\mathit` or even `\emph`. For example

  $$x^2 + y^2 = z^2 \qquad \text{compare with} \qquad \mathrm{x}^2 + \mathrm{y}^2 = \mathrm{z}^2$$

  A more aesthetic italic font in TEX is *math calligraphy*, it is often used as the name of general categories. For example,

  $$\text{a functor } F \text{ from } \mathcal{A} \text{ to } \mathcal{B}$$

  Its command is `\mathcal`.

- **Sans serif**, as its name, means that it has no serif. In Mathematics, it is often used as the name of some specific categories. For example,

  $$\mathsf{Grp} \qquad R\text{-}\mathsf{Mod} \qquad X\text{-}\mathsf{Sheaf}$$

  In TEX, sans serif is only permitted to be used in mathematical model, whose command is `\mathsf`.

- **Script** is created to write more fluently. In Mathematics, it is often used to represent a linear transform between linear spaces. For example

  $$\text{a linear transform } \mathscr{A} \text{ from } V \text{ to } W$$

  In TEX, sans serif is only permitted to be used in mathematical model, whose command is `\mathscr` using package `mathrsfs`.

- **Fraktur** is written easily by slanted pen but not friendly to modern pen. It is no doubts that it is also the most unrecognisable font in domain of mathematics. I will explain why the letter is written in that way afterwards. In mathematics, They stand for Lie algebras and ideals of a communicative rings, for example

$$x, y \in \mathfrak{g} \Rightarrow [x, y] \in \mathfrak{g} \qquad R/\mathfrak{a} \otimes_R R/\mathfrak{b} = R/(\mathfrak{a} + \mathfrak{b})$$

  In TEX, sans serif is only permitted to be used in mathematical model, whose command is `\mathfrak` using package `amssymb`.

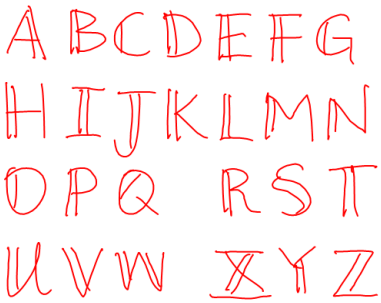
The font difficult to write down is the followings

$\mathbb{A}$(`\mathbb`) $\mathcal{A}$(`\mathcal`) $\mathscr{A}$(`\mathscr`) $\mathfrak{A}$(`\mathfrak`)

We will introduce the writing of them respectively.

2 mathbb

The font "Blackboard bold" is the most easy font to write (because it writes easy on blackboard).



$\mathbb{A}$ $\mathbb{B}$ $\mathbb{C}$ $\mathbb{D}$ $\mathbb{E}$ $\mathbb{F}$ $\mathbb{G}$

$\mathbb{H}$ $\mathbb{I}$ $\mathbb{J}$ $\mathbb{K}$ $\mathbb{L}$ $\mathbb{M}$ $\mathbb{N}$

$\mathbb{O}$ $\mathbb{P}$ $\mathbb{Q}$ $\mathbb{R}$ $\mathbb{S}$ $\mathbb{T}$

$\mathbb{U}$ $\mathbb{V}$ $\mathbb{W}$ $\mathbb{X}$ $\mathbb{Y}$ $\mathbb{Z}$

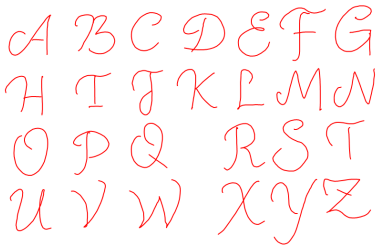
Figure 2: Blackboard bold

There are some tricks on writing these letters

- For almost of letters, they are just write it in normal way, and add a line as you like.
- On handwriting, you only need to "boldize" one part of the letter.
- For $\mathbb{A}$, someone is more like to write  .

3 mathcal

As I explained above, math calligraphy is a kind of Italic but of more aesthetic decorations. So to distinguish them, just add some decorations on the letters.



$$\mathcal{A}\ \mathcal{B}\ \mathcal{C}\ \mathcal{D}\ \mathcal{E}\ \mathcal{F}\ \mathcal{G}$$
$$\mathcal{H}\ \mathcal{I}\ \mathcal{J}\ \mathcal{K}\ \mathcal{L}\ \mathcal{M}\ \mathcal{N}$$
$$\mathcal{O}\ \mathcal{P}\ \mathcal{Q}\ \ \mathcal{R}\ \mathcal{S}\ \mathcal{T}$$
$$\mathcal{U}\ \mathcal{V}\ \mathcal{W}\ \ \mathcal{X}\ \mathcal{Y}\ \mathcal{Z}$$

Figure 3: math calligraphy

There are some remarks

- Writing E as an ε is a convention in calligraphy.
- When the begin of the letter is "|", one can decorate it into . This rule is suitable for $\mathcal{B}, \mathcal{F}, \mathcal{H}, \mathcal{R}, \mathcal{U}$.
- When the begin of the letter is "/", one can decorate it into  . This law is suitable for $\mathcal{A}, \mathcal{M}, \mathcal{N}$.
- When the begin of the letter is "\", one can decorate it into  . This rule is suitable for $\mathcal{V}, \mathcal{W}$.
- When the begin of the letter is "$\overline{\ }$", one can decorate it into  . This rule is suitable for $\mathcal{F}, \mathcal{I}, \mathcal{J}, \mathcal{T}$.
- When the begin of the letter is "$\frown$", one can decorate it into  . This rule is suitable for $\mathcal{C}, \mathcal{G}, \mathcal{S}$.
- When the begin of the letter is "D", one can decorate it into  . This rule is suitable for $\mathcal{D}, \mathcal{P}, \mathcal{R}$.

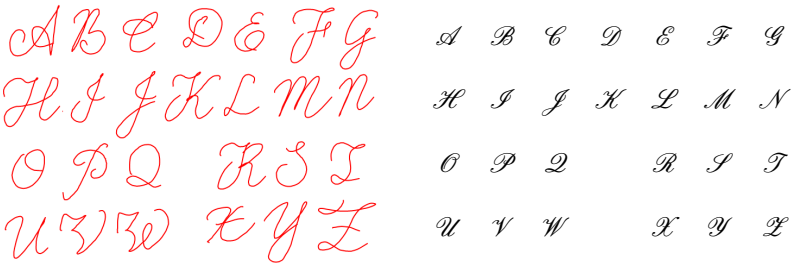


Figure 4: math calligraphy

4 mathscr

The Script is of more decorations.

The main reason most people cannot write it beautiful is the order of writing them. Script is designed to write more fluently, so there are some change is made in command order of writing letters to write the letter at once.

- The decoration is more exaggerated than `mathcal`,

Figure 5: more exaggerated decorations

- Writing $\mathscr{A}$, you should follow the write order

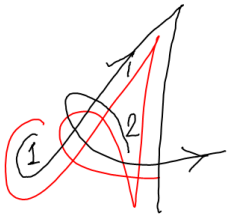


Figure 6: write order of writing $\mathscr{A}$

- There is another way to write $\mathscr{B}$, it is  . It is more like the typed one. I don't follow this because it cannot be written at once. The same to $\mathscr{R}$ and  .
- Written in one time is $\mathscr{C}$, the head of $\mathscr{C}$ is like writing a φ.
- Maybe it is amazing that $\mathscr{D}$ is also written at once. That means the "semi-circle" of $\mathscr{D}$ is written for button to head! But it is normal especially for Greek, because their Δ is just a triangulation of D.
- In calligraphy, E is always written in ε style.
- Easy to write is $\mathscr{F}$.
- Writing $\mathscr{G}$, you can imagine that you are writing a lowercase g with big head.
- Writing $\mathscr{H}$ is not too difficult.
- The begin of Writing $\mathscr{I}$ is from right to left. It is like writing the bass note (a highly easy-to-see notation of music, especially in staff), the small knot on the top is necessary.

Figure 7: $\mathscr{I}$ and bass note

- Writing $\mathscr{J}$ is like writing $\mathscr{I}$.
- Writing $\mathscr{K}$ is easy.
- Writing $\mathscr{L}$ is easy, imagining that you are writing a φ on the head.
- Writing $\mathscr{M}$ is easy.

- Writing $\mathscr{N}$ is easy.
- Writing $\mathscr{O}$ is easy.
- Writing $\mathscr{P}$ is like writing a long long f (without -) first and write a small D upwards.
- Writing $\mathscr{Q}$ is like writing a 2.
- Writing $\mathscr{R}$ is easy.
- Writing $\mathscr{S}$ is easy.

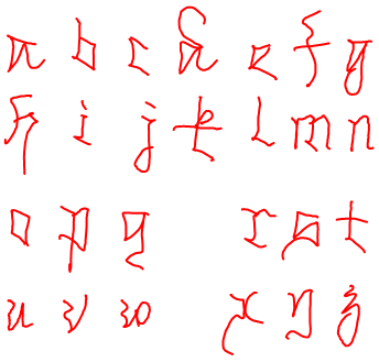
  you may complain that $\mathscr{S}$ is written in such confusing way, it is because it lean so much that an S becomes an f without $-$. The notation of integral $\int f(x)\mathrm{d}x$ is like this. And you must know sometimes Gauss is referred as "Gauß". The "ß" is not β; it is double s, the first s is prolonged (so-called long s) and the second connecting the first one.
- Writing $\mathscr{T}$ is easy. Do not confuse it with $\mathscr{J}$.
- Writing $\mathscr{U}$ is easy.
- Writing $\mathscr{V}$ is easy, if you have a good command of decoration mentioned above.
- Writing $\mathscr{X}$ is easy.
- Writing $\mathscr{Y}$ is easy.
- Writing $\mathscr{Z}$ is like write an α first and write a φ then, and write another α, add a $-$ last.

5 mathfrak

Fraktur is hard to recognize. I said that it is impossible or non-efficient to use modern pens to write them, this is especially right for Fraktur. But there are a lot of space using Fraktur in mathematics.

When you are writing papers or s'address lectures using blackboard, the situation force you to write them and distinguish them from other letters or same letter in other font.

The general rule writing the lowercase using pens is write them more "angular".



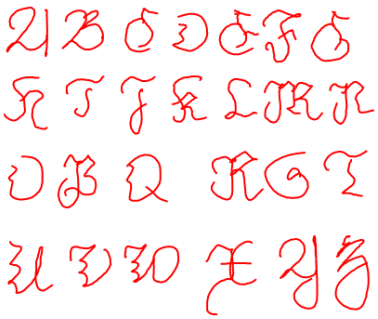
𝔞 𝔟 𝔠 𝔡 𝔢 𝔣 𝔤
𝔥 𝔦 𝔧 𝔨 𝔩 𝔪 𝔫
𝔬 𝔭 𝔮 𝔯 𝔰 𝔱
𝔲 𝔳 𝔴 𝔵 𝔶 𝔷

Figure 8: Fraktur-lowercase

- Writing 𝔞 is not difficult. I have see a lot of people writing them in the way the figure indicates.
- Writing 𝔟 is easy.
- Writing 𝔠 is easy.
- Writing 𝔡 is like writing a hook over an 𝔞. Actually, such 𝔡 is more like a ∂. (This way of writing 𝔡 is created by myself.)
- Writing 𝔢 is easy.
- Writing 𝔣 is easy.
- Writing 𝔤 is like writing an 𝔞 with a tail.
- Writing 𝔥 is easy.
- Writing 𝔦 should make serifs intentionally.
- Similar is 𝔧.
- Writing 𝔨 you should write a k firstly, and draw a small circle in the intersection point. Of course, write  is also reasonable.
- Writing 𝔪 is easy.
- Writing 𝔫 is easy.

- Writing $\mathfrak{o}$ is easy.
- Writing $\mathfrak{p}$ is easy.
- Writing $\mathfrak{q}$ is easy. (This way of writing $\mathfrak{q}$ (with small tail) is created by myself.)
- Writing $\mathfrak{r}$ is easy.
- The $\mathfrak{s}$ is like a angular 5.
- Writing $\mathfrak{t}$ is easy.
- The hard part of writing $\mathfrak{u}$ is the beginning, imagining you write a small 3 first.
- Similar is $\mathfrak{v}$.
- Similar is $\mathfrak{w}$. (I am not fully contend with such $\mathfrak{w}$ in TEX, it makes confusion with $\mathfrak{m}$ too much.)
- To write a $\mathfrak{x}$, write a inverse 5 first, and write a right bracket "(" tangent to the line.
- After thinking it as η, writing $\mathfrak{y}$ is easy.
- Writing $\mathfrak{z}$ is easy. In calligraphy, z is usually written as a 3.

The general rule of my program of writing the uppercase using pens is add them some sawtooth.



$\mathfrak{A}$ $\mathfrak{B}$ $\mathfrak{C}$ $\mathfrak{D}$ $\mathfrak{E}$ $\mathfrak{F}$ $\mathfrak{G}$

$\mathfrak{H}$ $\mathfrak{I}$ $\mathfrak{J}$ $\mathfrak{K}$ $\mathfrak{L}$ $\mathfrak{M}$ $\mathfrak{N}$

$\mathfrak{O}$ $\mathfrak{P}$ $\mathfrak{Q}$ $\mathfrak{R}$ $\mathfrak{S}$ $\mathfrak{T}$

$\mathfrak{U}$ $\mathfrak{V}$ $\mathfrak{W}$ $\mathfrak{X}$ $\mathfrak{Y}$ $\mathfrak{Z}$

Figure 9: Fraktur-uppercase

- First writing a 2 and an 1, you get a $\mathfrak{A}$.
- First writing a 2 and an 3, you get a $\mathfrak{B}$.
- Writing a 5 with long tail, then write a normal C, you get a $\mathfrak{C}$. The way I explained is the old mathematical Fraktur, you can see it in some not too old mathematical texts and papers.
- Writing $\mathfrak{D}$, first draw a 3 at left, and go around to get a $\mathfrak{D}$.
- Similiar is $\mathfrak{E}$ to $\mathfrak{C}$.
- Writing $\mathfrak{F}$, you should draw a $\sim$, and then write a "standing snake"

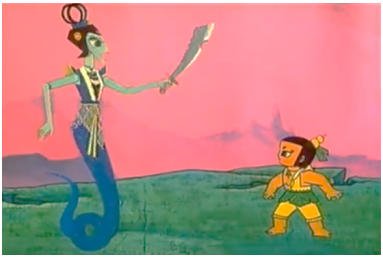


Figure 10: standing snake and Sanwa (01:57 of ep62918)

- Similiar is $\mathfrak{G}$ to $\mathfrak{C}$.
- Writing $\mathfrak{H}$ is easy, it is just big lower h.
- Writing $\mathfrak{I}$ is easy.
- Writing $\mathfrak{J}$ is easy.
- Writing $\mathfrak{K}$ is easy, it is just big lower h, and the different part from h can be consider as a square R.
- Writing $\mathfrak{L}$ is easy.
- Writing $\mathfrak{M}$ is divided into two step — writing a J and then writing an R.
- Writing $\mathfrak{N}$ is easy.

- Writing $\mathfrak{O}$ is easy, first draw a 3 to be sawtooth.
- Writing a big J and write a 3, you get $\mathfrak{P}$.
- It is similar to write $\mathfrak{Q}$ compared with $\mathfrak{O}$.
- Writing $\mathfrak{R}$ is easy.
- The start point is in the middle of $\mathfrak{S}$, the rest part is written by drawing around. You may confused in why it becomes such a letter which looks like G if do not know the rule of fraktur.

Figure 11: How it becomes $\mathfrak{S}$

  The same reason for σ, although you may write σ as a 6 lying down or a 5 with no neck.

- Writing $\mathfrak{T}$ is easy.
- Writing $\mathfrak{U}$ is similar to $\mathfrak{D}$ — adding 3.
- Similarly, writing $\mathfrak{V}$ is also adding 3.
- Similarly, writing $\mathfrak{W}$ is also adding 3.
- Writing $\mathfrak{X}$ is easy.
- Writing $\mathfrak{Y}$, write a 2 first.
- Writing $\mathfrak{Z}$ is easy.

6 Declaration

The above fashions to writing is only my programme, you may dislike it, but I just want to give some suggestions. And of course, calligraphy needs exercise beaucoup. It you do not suffer to do it, you must become a good font user in mathematics. Otherwise, you use less special fonts, that's all. The handwriting involves anything but mathematical level.