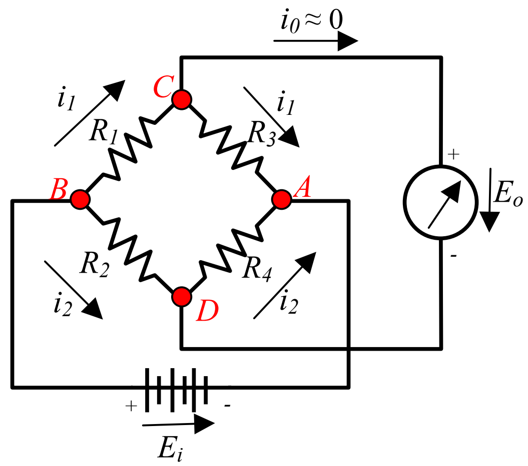
Wheatstone Bridge Calibration for Strains

Background

The strain lab uses two strain gages mounted on the top and bottom surface of a beam in bending to measure the surface strain due to bending. In the process of calibrating the bridge a calibration resistor is used to **relate a known resistance change to an equivalent strain**. This document describes the theoretical background behind this calibration process.

Wheatstone Bridge

The Wheatstone bridge consists of 4 resistors connected in a configuration as shown on the right. Across two opposite connection points (A and B) a voltage E_i is applied (also called the excitation voltage).



Wheatstone Bridge

We can make the following assumption:
$i_0 \approx 0$ (high impedance voltmeter)

Using Kirchhoff's 2nd law, which states that the sum of the voltage in a closed loop has to be zero, we can write for the loop, which goes from the excitation voltage through resistors 2 and 4:

$E_i - (R_2 + R_4)i_2 = 0$, therefore

$$i_2 = \frac{E_i}{R_2 + R_4}$$

For the loop going through resistors 1 and 3 we get

$E_i - (R_1 + R_3)i_1 = 0$; therefore $i_1 = \dfrac{E_i}{R_1 + R_3}$

From these equations the unknown currents have been determined. However, we are interested in the voltage difference between points C and D (the output voltage of the bridge, E_0), which we want to relate to resistance changes in the bridge. This can be found from a 3rd voltage loop across resistors 3 and 4 and E_0.

$$E_0 + R_4 i_2 - R_3 i_1 = 0$$

Solving for the output voltage and substituting the earlier results for the currents gives an equation for the bridge output voltage as a function of the resistors and the input excitation voltage E_i.

$$E_0 = \left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right] \bullet E_i \quad \text{or} \quad \boxed{\frac{E_0}{E_i} = \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}}$$

This equation can be used to determine, how a resistance change in the bridge changes the output voltage.

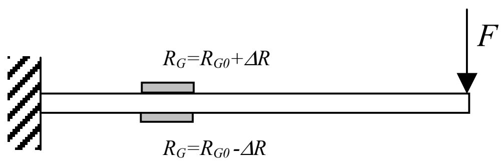
A **"balanced bridge"** has zero output voltage. Balancing is achieved by choosing resistors that make the expression $\frac{R_3}{R_1+R_3}-\frac{R_4}{R_2+R_4}$ go to zero.

This is equivalent to $\frac{1}{\frac{R_1}{R_3}+1}-\frac{1}{\frac{R_2}{R_4}+1}=0$ or $\frac{R_1}{R_3}=\frac{R_2}{R_4}$

Wheatstone Bridge with two strain gages

The Wheatstone bridge is now modified so that resistor 1 and 2 are strain gages. Remember that a strain gage produces a resistance change proportional to strain. The proportionality factor G_F is called the gage factor and depends on the strain gage material.

$$\frac{\Delta R}{R}=G_F\frac{\Delta L}{L}=G_F\,\varepsilon$$



We can therefore write for the resistance change in the top and bottom strain gage

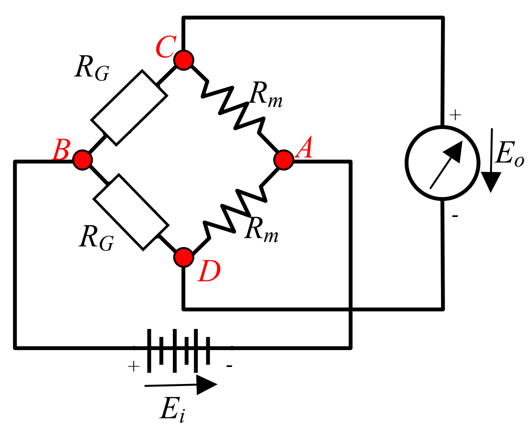
$$R_1=R_{G0}+\Delta R=R_G(1+G_F\,\varepsilon)$$
$$R_2=R_{G0}-\Delta R=R_G(1-G_F\,\varepsilon)$$

where R_{G0} is the nominal (unstrained) gage resistance. Substitution of these results into the equation for the bridge output voltage yields

$$\frac{E_0}{E_i}=\frac{R_m}{R_{G0}(1+G_F\,\varepsilon)+R_m}-\frac{R_m}{R_{G0}(1-G_F\,\varepsilon)+R_m}$$

where R_m is the nominal value of R_3 and R_4. This equation is non-linear in the strain ε. However, for small strains, we can linearize this equation around ε using a Taylor Series.

$$\boxed{\frac{E_0}{E_i}\approx-\frac{R_mR_{G0}}{(R_{G0}+R_m)^2}2G_F\varepsilon}$$



Wheatstone Bridge with two strain gages

where we only kept the term linear in ε. Higher order terms are small because of the small strains. If we choose the resistors in the Wheatstone bridge to have the same nominal resistance as the strain gages,

$R_m = R_{G0}$,

the expression simplifies even more to

$$\frac{E_0}{E_i}\approx-\frac{G_F}{2}\varepsilon \qquad (for\;\; R_m=R_{G0})$$

Therefore for a strain gage bridge with **nominal resistors being identical on all 4 arms**, the ratio of output to excitation voltage will be simply the strain * gage factor / 2. Note that in the lab the 4 arms do not have the same resistance.

Note that since typical strains are small and excitation voltages have to be kept low to manage current flow and power dissipation in the circuit, very small voltages have to be measured reliably.

Lab Bridge configuration

In the lab a commercial bridge amplifier is used. It has adjustable resistors to balance the bridge, calibration resistors and an amplifier to increase the magnitude of the bridge output.

The wiring diagram of the bridge circuit is shown on the right. In order to determine the equivalent resistance R_m in this circuit for branch A-C and A-D, we have to add the resistors shown.

For a balanced bridge with unstrained strain gages we know that ½ of the adjustable 200kΩ resistor is on either side of the bridge circuit arms A-C and A-D.

Following the rules for resistor addition we get

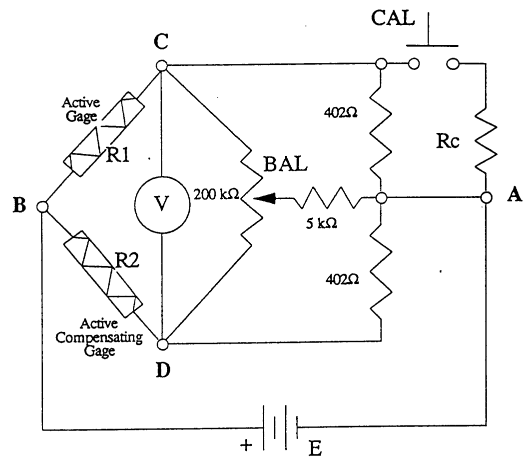
$$R_m = \left(\frac{1}{402.5} + \frac{1}{100{,}000 + 5{,}000} \right)^{-1} = 401.0\,\Omega$$

The strain gage has a nominal value of

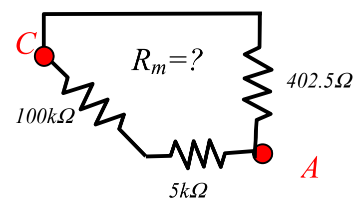
R_{G0}=120.5Ω

and a gage factor

G_F=2.09



Wheatstone Bridge Configuration in Lab



Resistor Detail (A to C)

Using the results from the previous section we have

$$\frac{E_0}{E_i} = -\frac{R_m R_{G0}}{(R_{G0} + R_m)^2} 2 G_F \varepsilon = -\frac{401\Omega \cdot 120.5\Omega}{(120.5\Omega + 401\Omega)^2} \cdot 2 \cdot 2.09 \cdot \varepsilon = -0.7427\,\varepsilon$$

The above equation establishes the relationship between strain ε, bridge excitation voltage E_i and measured output voltage E_0.

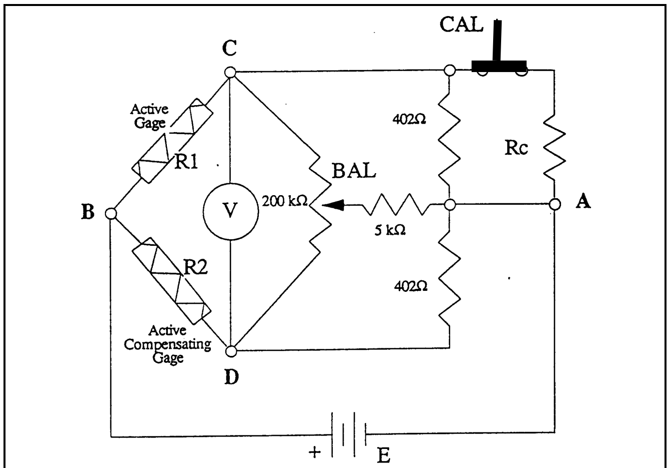
The adjustable bridge amplifier in the BAM further amplifies E_0 with an undetermined gain factor. What is eventually measured with the data acquisition is E_{DAQ} , where $E_{DAQ} = G_{Ampl}\, E_0$, where G_{Ampl} is the amplifier gain.

Lab Bridge configuration with Calibration Resistor

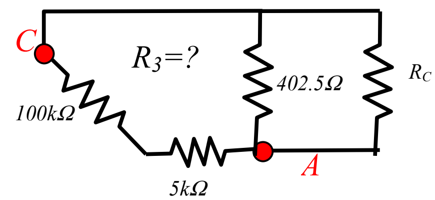
When the calibration (CAL) button is pressed, an additional resistor is added to the A-C branch of the Wheatstone bridge.

The wiring diagram of the bridge circuit is shown on the right but now with the CAL button pressed. This changes the equivalent resistance R_3 in the A-C branch circuit only but not R_4 in the A-D branch.

The imbalance in the bridge between A-C and A-D will produce an output voltage E_0.



Wheatstone Bridge Configuration in Lab



Resistor Detail (A to C) with R_c

From before we know that

$$R_4 = R_m = 401.0\Omega$$

Following the rules for resistor addition for the A-C branch we get

$$R_3 = \left(\frac{1}{R_m} + \frac{1}{R_C}\right)^{-1}$$

Again we make use of the fact that the calibration resistor will be considerably larger than R_m. Therefore we can do a linear Taylor Series expansion to approximate R_3.

$$R_3 = \left(\frac{1}{R_m} + \frac{1}{R_C}\right)^{-1} \approx R_m\left(1 - \frac{R_m}{R_C}\right)$$

The equation for the output voltage becomes

$$\frac{E_0}{E_i} = \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} = \frac{R_m\left(1 - \frac{R_m}{R_C}\right)}{R_{G0} + R_m\left(1 - \frac{R_m}{R_C}\right)} - \frac{R_m}{R_{G0} + R_m}$$

The calibration resistor is large compared to the bridge resistance, therefore we can assume that $\frac{R_m}{R_C} << 1$. As a shortcut we write $\rho = \frac{R_m}{R_C}$. The Taylor series approximation for the bridge output E_0 in terms of the normalized (small) calibration resistor ratio ρ then becomes

$$\boxed{\frac{E_0}{E_i} \approx -\frac{R_m R_{G0}}{(R_{G0} + R_m)^2}\rho = -\frac{R_m R_{G0}}{(R_{G0} + R_m)^2}\frac{R_m}{R_C}}$$

This equation can be used to determine the output voltage E_0 for known nominal bridge R_m and gage resistance R_{G0} and known calibration resistor R_C.

Comparison of Bridge Output due to strain vs calibration resistor

So far we found that the bridge output voltage E_0 varies with strain according to

$$\frac{E_0}{E_i} \approx -\frac{R_m R_{G0}}{(R_{G0}+R_m)^2} 2G_F \varepsilon \qquad \text{bridge output to strain}$$

and if no strain is applied but a calibration resistor is inserted into the A-C branch of the bridge

$$\frac{E_0}{E_i} \approx -\frac{R_m R_{G0}}{(R_{G0}+R_m)^2} \frac{R_m}{R_C} \qquad \text{bridge output to calibration resistor}$$

The next question we ask is, **what kind of strain ε_{cal} would be required to produce the same output E_0 that we get by adding the calibration resistor to the circuit**. Note that in the lab we also have an amplifier with adjustable gain, but the amplification is the same whether the bridge voltage occurs due to strain or a calibration resistor. Therefore we can compare the voltages either before or after amplification. We set

$$E_0^{Strain} = E_0^{CalResistor}$$

which (since the excitation voltage is constant) is equivalent to

$$\frac{E_0^{Strain}}{E_i} = \frac{E_0^{CalResistor}}{E_i}$$

Therefore

$$-\frac{R_m R_{G0}}{(R_{G0}+R_m)^2} 2G_F \varepsilon_{Cal} = -\frac{R_m R_{G0}}{(R_{G0}+R_m)^2} \frac{R_m}{R_C}$$

Solving for the strain as a function of the calibration resistor we get

$$\boxed{\varepsilon_{Cal} = \frac{R_m}{2G_F R_C}}$$

This strain, ε_{cal} , is the strain which is equivalent to an applied calibration resistor of magnitude R_C.

In the lab the nominal cal resistor (C=1) is R_C=1MΩ, G_F=2.09 and R_m was determined to be 401Ω. Therefore the equivalent strain is

$$\varepsilon_{Cal} = \frac{401\Omega}{2\cdot 2.09\cdot 10^6\Omega} = 95.9\cdot 10^{-6}\, {}^{in}\!/_{in}$$

The table in the lab notebook lists a value of $96.1 \cdot 10^{-6}$, which more closely corresponds to R_m=402.5Ω. But this is a minor difference (0.2%).

For different settings of the CAL switch (C), different cal resistors can be engaged. The general expression for the BAM is

$$\boxed{R_C = \frac{1M\Omega}{C}}$$

which can be used to determine the equivalent strain based on the setting of the switch. A corresponding table for different values of C is given in the lab manual.