Writing for computer science: Chapter 5: Mathematics

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Introduction

Mathematics gives solidity to abstract concepts.

Mathematical notation can be used to describe algorithms, data structures, or just about any of the objects that com-puter scientists study.

Clarity: Mathematical writing is essential to be precise

Tip 1: Many discussions can be clarified through the use of mathematical notation.

Bad example:

An inverted list for a given term is a sequence of pairs, where the first element in each pair is a document identifier and the second is the fre- quency of the term in the document to which the identifier corresponds.

Good example:

An inverted list for a term t is a sequence of pairs of the form $\langle d, f \rangle$, where each *d* is a document identifier and *f* is the frequency of *t* in *d*.

Clarity: Mathematical writing is essential to be precise

Tip 2: Many terms are confusing and should be used carefully.

Example:

- normal \Rightarrow usual
- formula \Rightarrow equation
- equivalent \Rightarrow similar
- element \Rightarrow partition
- average \Rightarrow mean
- metric ⇒ measure

Theorems

Tip 3: Steps in the logic of a proof should be simple, the gaps can be completed by a reader mechanically.

Tip 4: Theorems, definitions, lemmas, and propositions should be numbered, even if there are only two or three of each in the paper.

Tip 5: Some presentation problems are not easily resolved. One approach is to state the main theorem first, then state and prove the lemmas before giving the main proof.

Tip 6: Mathematics is usually presented in italics. For example: "of length *n*"

Tip 7: Use the same font for the same variable.

Tip 8: Parentheses should stand out from the expression they enclose:

 $\times (p \cdot (\sum_{i=0}^{n} A_i))$ $\checkmark (p \cdot (\sum_{i=0}^{n} A_i))$

Tip 9: Mathematics should not be used at the start of a sentence:

× $p \leftarrow q_1 \land \cdots \land q_n$ is a conditional dependency.

✓ The dependency $p \leftarrow q_1 \land \cdots \land q_n$ is conditional.

Tip 10: Give the type of each variable every time it is used. Watch out the misplaced types of variables:

 \times The values are represented as a list of numbers *L*.

 \checkmark The values are represented as a list *L* of numbers.

Tip 11: Mathematics should not take the place of text, readers may get lost.

$$\begin{array}{ll} \mathsf{X} \quad \mathrm{Let} \ \langle S \rangle = \left\{ \begin{array}{l} \sum_{i=1}^{n} \alpha_{i} x_{i} \mid \alpha_{i} \in F, 1 \leq i \leq n \end{array} \right\}. \quad \mathrm{For} \ x = \sum_{i=1}^{n} \alpha_{i} x_{i} \ \mathrm{and} \ y \\ = \sum_{i=1}^{n} \beta_{i} x_{i}, \ \mathrm{so} \ \mathrm{that} \ x, y \in \langle S \rangle, \ \mathrm{we} \ \mathrm{have} \ \alpha x + \beta y = \alpha \left(\sum_{i=1}^{n} \alpha_{i} x_{i} \right) + \\ \beta \left(\sum_{i=1}^{n} \beta_{i} x_{i} \right) = \sum_{i=1}^{n} \left(\alpha \alpha_{i} + \beta \beta_{i} \right) x_{i} \in \langle S \rangle. \end{array}$$

Tip 11: Mathematics should not take the place of text, readers may get lost.

✓ Let $\langle S \rangle$ be a vector space defined by

$$\langle S \rangle = \left\{ \sum_{i=1}^n \alpha_i x_i \mid \alpha_i \in F, 1 \le i \le n \right\}.$$

We now show that $\langle S \rangle$ is closed under addition. Consider any two vectors $x, y \in \langle S \rangle$. Then $x = \sum_{i=1}^{n} \alpha_i x_i$ and $y = \sum_{i=1}^{n} \beta_i x_i$. For any constants $\alpha, \beta \in F$, we have

$$\begin{aligned} \alpha x + \beta y &= \alpha \left(\sum_{i=1}^{n} \alpha_{i} x_{i} \right) + \beta \left(\sum_{i=1}^{n} \beta_{i} x_{i} \right) \\ &= \sum_{i=1}^{n} \left(\alpha \alpha_{i} + \beta \beta_{i} \right) x_{i} , \end{aligned}$$

so that $\alpha x + \beta y \in \langle S \rangle$.

Tip 12: Mathematical expressions should not run together.

- × For each x_i , $1 \le i \le n$, x_i is positive.
- ✓ Each x_i , where $1 \le i \le n$, is positive.

Tip 13: Consider breaking down expressions to make them more readable, especially if doing so enlarges small symbols.

×
$$f(x) = e^{2^{-\frac{b}{a}x\sqrt{1-\frac{a^2}{x^2}}}}$$

<

Ranges

Tip 14:

The closed range of real numbers *r*, where $a \Box r \Box b$ is represented by "[a, b]"

The open range $a \le r \le b$ is represented by "(a,b)"

The expression $1 \square i \square 6$ should be replaced by i = 1, 2, ..., 6 if it is not clear that *i* should be integer.

Alphabets

Tip 15: Most readers are familiar with only a few Greek letters, so use of unfamiliar letters should be minimized.

Tip 16: Some alphabets can cause confusion.

Symbol		Confused with	
Е	epsilon	е	
η	eta	n	
ι	iota	i	
μ	mu	u	
ρ	rho	р	
υ	upsilon	ν	
ω	omega	w	
V	or	v	
×	proportional	α	alpha
Ø	empty set	$oldsymbol{\phi}$	phi

Line breaks

Tip 17: Avoid letting a number, symbol appear at the start of the line, particularly if it the end of a sentence.

- X We have therefore introduced an additional variable, denoted by *x*. It allows ...
- ✓ We therefore introduce an additional variable, denoted by x. It allows ...
- X Accesses to the new kind of disk typically require about 12 ms using our techniques.
- ✓ Accesses to the new kind of disk typically require about 12 ms using our techniques.

Percentage

Tip 18: Percentages should always be in figures.

Bad Example:

The increase was over 5 per cent.

Good Example:

The increase was over 5%.

Tip 19: Do not mix modes.

- × There were between four and 32 processors in each machine.
- \checkmark There were between 4 and 32 processors in each machine.
- × The sizes were 7.31 Kb and 181 Kb, respectively.
- \checkmark The sizes were 7.3 Kb and 181.4 Kb, respectively.

Units of measurement

Tip 20: Units should be typeset in the font used in the paper for text, even when they are part of a mathematical expression.

- × The volume is $r^p Kb$ in total.
- ✓ The volume is r^p Kb in total.